APPLYING COMPLEX NETWORKS TO PAVEMENT MANAGEMENT SYSTEMS IN THE MEXICAN ROADWAY

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ABSTRACT

The purpose of the analysis is to quantify large-scale statistical properties of complex networks at the Mexican road system in the year of 2005 in order to use them into the pavement management system (PMS), specifically identifying and classifying types of pavement networks. The application of such properties reveals important information about the structure of the roadway resulting in an effective database and a better rational decision about the best budgeting allocation.

The analysis presents three groups of measures that describe statistical characteristics of complex networks related to topological and geometric variations. The first group covers classic measures related to graph theory, the second group emphasizes the importance of heterogeneity in road segments, and the third group computes connectivity levels and displays vector maps related to geometry patterns.

The results show an effective way to define pavement networks and identify Maintenance and Repair (M&R) priorities. Therefore, the application of complex networks to the PMS can help private and public agencies to reduce time and avoid costly errors related to maintain and manage pavement networks.

1. INTRODUCTION

The application of complex networks in the analysis of road systems is a relative new approach in the field of transport networks, where large scale properties are computed in order to understand the performance of the system. Road systems are particularly interesting because they are represented by networks with nodes and edges constrained by geographic environments, for example nodes exemplify intersections and origin and destination points, and edges correspond with segment of roads. Furthermore, they exhibit different properties compared with similar systems, for example the distribution of node degree in the airline system.

On the other hand, the pavement management system (PMS), defined as a process to store and analyze pavement information of roadways in order to inform and prioritize cost-effective Maintenance and Repair (M&R) strategies, has been applied by managers and engineers to preserve the road infrastructure [2, 19]. Even though there is a long tradition applying this process, the complexity of current road networks makes difficult the successful application of it, for example Shahin [23] points out that "pavement networks must now be managed, not simple maintained." This phrase emphasizes a selection problem of M&R techniques related to incomplete information about different pavement requirements on the network. Then, the PMS needs to add different approaches in order to determine M&R needs and priorities.

Therefore, the application of complex networks based on geospatial data, for example shapefiles, to the PMS can provide effective tools to collect, update, and maintain priority pavements. With this in mind, the analysis is centered to the network definition, which is the starting point of the PMS process and supports other data collection and analysis. Then, the relationship between complex networks and the PMS is exemplified using data of the Mexican road system.

The Mexican road system exhibits significant properties compare with airlines, rails, and maritime transportation systems. Such a system is the most important network in Mexico because it connects completely all urban areas and carries most of the goods movements and people trips, for example 57% of cargo and 97% of passenger trips flowed by this system in the year of 2009 [17]. Furthermore, the public and private investment in roads of the sector has increased 55% from 2008 to 2009 [21]. Under those circumstances, we can say that the road system is the core of the transportation in the country.

The complex network perspective is used to have an ample picture of the Mexican road system analyzing topological and geometrical attributes. In addition to transport geography, economics, and urban planning fields, complex networks have been applied to study transport systems, where methods and techniques, which were developed by scientists of different fields, describe, compute, and simulate large scale features of such systems.

For example, structural properties of complicated road systems are analyzed by Xie and Levinson [29] and dynamics processes are studied by Barabási [4], Newman [20], and Xie and Levinson [30].

The analysis presents three groups of complex network measures that describe structural characteristics by statistical values. The first group covers classic measures related to the graph theory, specifically the number of nodes and edges, diameter of the network, average path length, transitivity, average clustering coefficient, β and γ indices, and the number of connected components. Previous studies related to these measures are Garrison [9], Kansky [13], and Xie and Levinson [30]. The second group reports measures of heterogeneity, which are computed by an edge degree distribution and a statistical collective measure of entropy, where similar studies are Shannon [24], Balch [3], Sole and Valverde [27], Newman [20], Albert, Jeong, and Barabási [1], and Xie and Levinson [29]. The third group computes connectivity measures related to a Cyclomatic number, α index, and circuits and trees ratios. Illustrations of these measures are Hargett and Chorley [12], Gibbons [10], Marshall [18], Levinson [15], and Xie and Levinson [30].

The information is based on open access information related to a vector map of roads revised in the web page of the National Institute of Statistics and Geography of Mexico. Based on the scale of the analysis, such a map was modified by a Douglas-Peucker algorithm with a threshold equals to 80, where this algorithm is a line simplification process that reduces the complexity of the vector features without changes its topology [11]. In other words, the simplification process reduces the geometry of each line segment preserving the origin and destination points, for example a line formed by five points and four lines can be reduced by three points and two lines. Furthermore, the analysis was programmed in Python applying several modules for scientific computing, for example OGR, Networkx, and Numpy.

The structure of the analysis is divided into five sections. The first section describes the process to define a pavement network used in the PMS framework. The second section explains briefly complex network measures. The third section show the computation of such measures identifying and classifying geometric structure of the Mexican road system. The fourth section exemplifies the application of complex networks to the PMS defining and prioritizing pavement networks at Mexico. The last section concludes and makes some recommendations.

2. NETWORK DEFINITION PROCESS

The application of the PMS follows a systematic approach that can be divided into six steps: network or inventory definition, pavement inspection, condition assessment, condition prediction, condition analysis, and working plan [23]. Despite the fact that each of these steps are essential for the PMS, this analysis considers only the network or inventory definition because it establishes the actual knowledge of the system, stores current available information, sets initial conditions, and guides data analysis in the PMS.

In addition, the network definition process is divided in network, branch, and section identification in order to define and classify types of pavement networks (Figure 1). Then, network identification is the process to establish a pavement structure describing the scale of the analysis based on types of surfaced and unsurfaced facilities, for example a regional roadway network related to metropolitan areas [17].

Branch identification is a division of the pavement network giving a logical description of classes of networks based on distinct uses, for example roads based on similar functional and operational characteristics can be classified by levels of accessibility or mobility services, streets or arterial roads respectively [29].

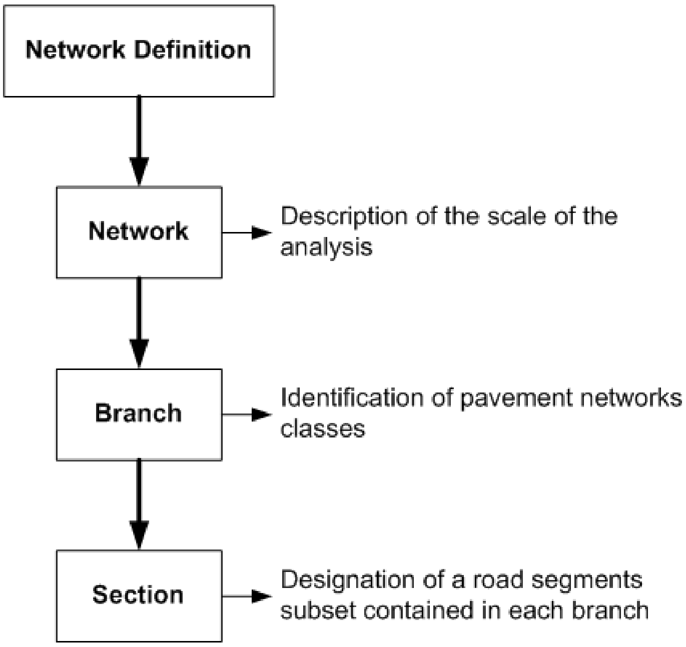


Figure 1 – Network Definition Process

Section identification is a subset of segments of roads contained in each branch having consistent characteristics related to areas and lengths. Based on the work of Shahin [23], there are many factors to be considered in this part of the analysis, for example pavement structure corresponding to thickness and materials composition; construction history describing years, contractors, and techniques; traffic taking into account the volume and load intensity; pavement rank showing the change in traffic; drainage facilities and shoulders counting the number of these provisions; condition considering changes in distress types, quantities, and causes; and size reflecting the economic impact of selecting different length of sections.

3. COMPLEX NETWORK MEASURES

After describing the process of pavement network definition, the next step is to explain complex networks measures in order to quantify, analyze, and show specific information that help to identify and determine M&R needs and priorities.

A road network is defined as a planar graph *G* conforming by nodes and edges of the form *G = {V, E}*, where *V* is a collection of nodes (vertices), and *E* is a group of undirected links (edges) that connect nodes. Furthermore, if *G* is unconnected, it can consist of connected sub-graphs, where each of them is a fully connected graph and every node is undirected connected to every other node [26]. These sub-graphs are known as connected components of *G* [20, 30].

Based on topological and geometric variations, complex network measures are divided into three groups (Figure 2).

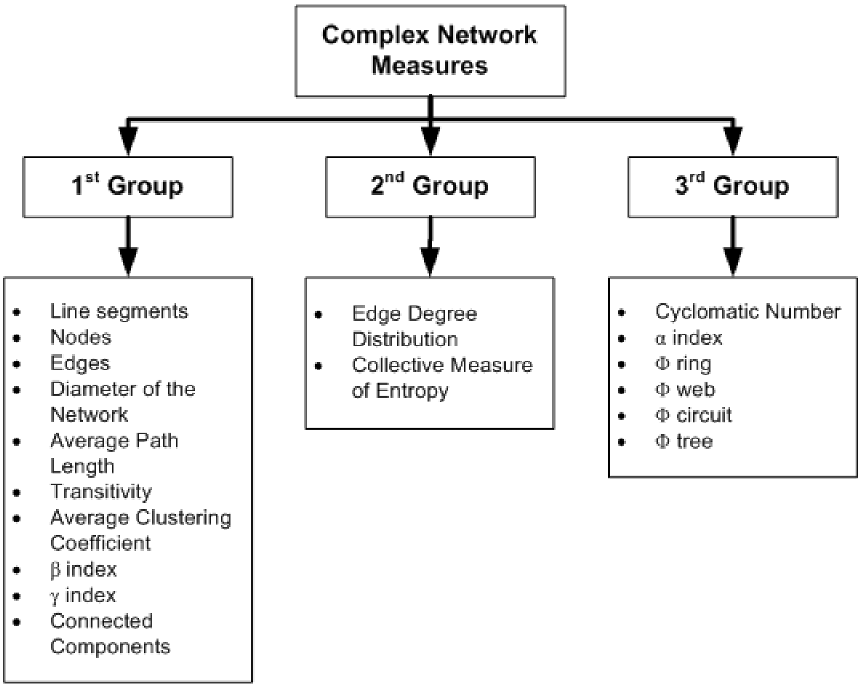


Figure 2 – Complex Network Measures

The first group is related to topological measures of planar networks based on three main parameters: the number of nodes, edges, and connected components. Using these parameters, it is possible to compute the diameter of the network, average path length, transitivity, average cluster coefficient, β and γ indices, and the number of connected components [20].

The diameter of the network represents the maximum distance from two nodes that are far away, in other words, it is the maximum length or number of edges between these nodes. In this case, such a measure represents how many intersections and origin and destination points are between the farthest nodes in the road system.

Average path length or average shortest path length represents the number of nodes along the shortest path for all possible pairs of nodes in the network.

This measure is related to efficiency levels of the network meaning how fast is traveling from one node to another.

The transitivity measure means the presence of a large number of triangles, which are sets of three nodes each of which is connected to each of the others. In road networks, this value corresponds to a probability of two roads intersecting them in a node and reflects the level of redundancy in the network, for example a small value describes a low level of redundancy. In the same way, the average clustering coefficient measures the presence of a large number of triangles per node. The difference between those measures is that the average clustering coefficient computes the mean of the fraction of possible triangles that exist for each node, and the transitivity computes the ratio of the means [20].

The β index measures the level of connectivity in a graph, where it computes the ratio between the number of edges and the number of nodes. Values higher than one are related to complex networks. Likewise, the γ index quantifies the level of connectivity based on the relationship between the number of observed edges and the maximum number of possible edges in the network. This value is between cero and one, where values closer to one indicates a more connected network and values closer to zero suggests an unconnected graph.

Finally, the number of connected components counts the number of sub-graphs in G.

The second group of measures is related to the concept of heterogeneity. In road networks, heterogeneity represents a level of different functional and operational properties related to segments of roads [29]. An edge degree distribution and a collective measure of entropy are computed in order to show how complex is the road system.

An edge degree distribution can be defined as the number of adjacent edges connected to one edge by its origin and destination points. In this case, the distribution shows a histogram based on road segments, where the x axis indicates bins related to the number of adjacent edges, and the y axis shows the probability that an edge corresponds to some bin. Therefore, this analysis determines the probability of any edge that is connected to the others and identifies the type of statistical distribution. Because road networks exemplify real world random networks, the distribution of node degree follows a normal distribution, that is each edge has a limited number of connections and lengths [4, 5, 8, 16, 29].

On the other hand, the collective measure of entropy *H* computes the level of disorder or heterogeneity in the system. In order to compute this measure, edges are considered as independent agents and grouped into subsets based on their road properties related to their functional classification [3, 29]. In this case, the classification emphasizes the mobility characteristic based on the number of lanes per road, for example two, four, and more lanes. Therefore, the entropy is calculated as the sum of the frequency of edges in the subset over the total number of edges [3, 7, 29]. If the value of entropy is equal to zero, it represents a homogeneous network; and if the value is higher than zero, it indicates heterogeneity in the network. Large values of entropy suggest greater levels of heterogeneity in the system.

The third group of measures computes and identifies levels of branching and circuit structures, which reflect more precise measures of connectivity and geometry patterns, based on the Cyclomatic number, α index, and circuits and trees ratios.

The Cyclomatic number indicates the number of independent cycles in the network, where each cycle starts and ends in the same node. When such a number is larger than zero, there is at least one circuit in the network [29]. Furthermore, the α index computes the ratio between the actual number of circuits in the network and the maximum number of them indicating the proportion of circuits in the network.

In addition, circuit and tree ratios evaluate the performance of the network geometry. Based on the work of Xie and Levinson [29] a circuit structure can be defined as a bidirectional closed path that begins and ends at the same point, and trees structures can be specified as a set of connected lines without any complete circuit. Therefore, the circuit ratio is computed as the sum of the participations of rings and webs structures in the main connected component, where a ring is defined as a circuit block (block that contains at least one circuit and incorporates neither bridges nor articulation points) holding only one circuit, and a web is specified as a circuit block incorporating more than one circuit block. Consequently, a tree ratio is computed as the difference between the value of one and the circuit ratio. These ratios range from zero to one and point out the level of arterials roads connected as circuits and branching structures connected as trees. Therefore, between both structures, arterial roads are considered as one of the most important networks because they provide the highest levels of mobility in long uninterrupted distance, with some degree of access control, and connectivity to significant urban and rural areas [28].

4. STRUCTURE OF THE MEXICAN ROADWAY

Complex network measures are applied to the case of the Mexican road system identifying topological and geometrical large-scale network characteristics. Then, the first group of measures presents an overview of topological properties based on main network parameters (Table 1).

Comparing the number of line segments, nodes, and edges with the original map of the national road network at Mexico, line segments remain the same with a value of 18,917, and nodes and edges decrease because of the line simplification algorithm, from 537,748 to 24,644 nodes and from 540,220 to 27,026 edges. Even thought such a simplification, these values represent a complex structure because of the large amount of information related to the network. Next, the diameter of network has a value of 802 representing the maximum length or number of edges between nodes that are far away. In essence, in order to travel from two distant nodes, there are 802 intersections between them. This long diameter characterizes planar networks.

Table 1 - Properties of the Road System in Mexico

Properties	Value
Line segments	18,917
Nodes	24,644
Edges	27,026
Diameter of network	802
Average path length	190
Transitivity	0.0096
Average clustering coefficient	0.0045
β index	1.096
γ index	0.3655
Connected Components	1

On the other hand, the average path length takes a value of 190, which is very high compare with scale-free networks, representing the number of nodes along the shortest path for all possible pairs of nodes in the network. Likewise the preview measure of diameter, large values are expected because of the network topology. Moreover, the value of transitivity and the average clustering coefficient are very small and towards to zero meaning a low level of redundancy in the system. In other words, traveling between any two nodes, it is very difficult to find roads forming a triangle.

Later on, the β and γ indices, as well as the number of connected components, represent levels of connectivity in the system. Based on the value of the β index, edges are confirmed as the predominant feature in the network. On the other hand, the value of 36% of the γ index represents a low connectivity in the system, that is, it is possible to create new connections between line segments. Finally, the number of connected components is one corresponding to one sub-graph in G, where all nodes are connected to edges, and no more information can be added. This result is a common property in real road systems.

The second group of measure analyzes the complexity of the system applying an edge degree distribution and a collective measure of entropy. Even though road systems do not present common features related to small-world networks, for example the existence of the power law degree distribution in nodes, they have other properties that define such a system as complex. These properties are fundamentally related to edges, which have specific functional properties and operational performance, and can be explained by the concept of heterogeneity. Therefore, heterogeneity means a hierarchy of roads that exist without a prespecified design [29].

The edge degree distribution in the system is analyzed by a histogram and by the Kolmogorov-Smirnov (KS) test for normality [14, 25]. As a result, the analysis confirms a normal distribution (Figure 3).

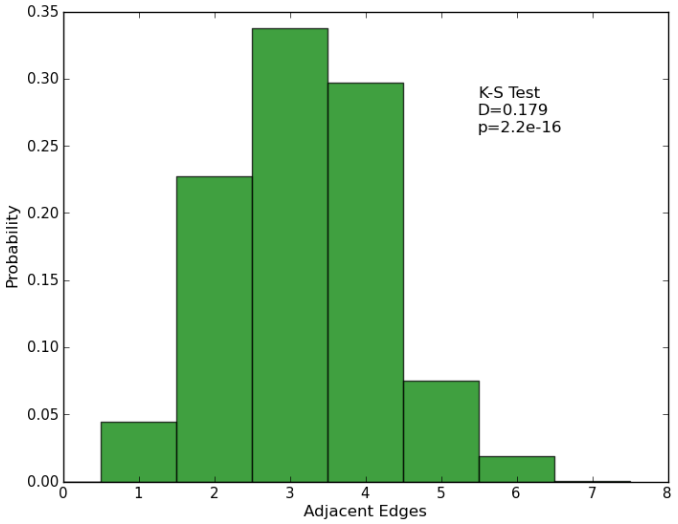


Figure 3 - Edge Degree Centrality Distribution

The first column represents the probability to find dead-end roads in the network, where its value is approximately 0.4%. The next column corresponds to line segments that are connected to two edges, for example two roads are linked to other in the origin and destination nodes. The probability of such segments is equal to 23%. The third column is the mean value of the distribution and shows a value of 34% of edges that are linked to three others. Next column presents a probability of 30%, which means the ratio of probable roads that have four adjacent line segments in the system. The fifth, sixth, and seventh columns represent the number of adjacent roads related to five, six, and seven segments of roads, their values are equal to 8%, 2%, and less than 1% respectively. As a result if a line segment increases its number of adjacent edges, there is a high probability than that segment presents traffic problems and, then, pavement surface distresses.

On the other hand, the collective measure of entropy H is equal to 0.972, which considers roads as a collection of agents grouping into subsets based on their functional classification. Such a classification emphasizes the mobility characteristic based on their number of lanes: one, two, four, six, and more than six lanes. Therefore, this value means the presence of entropy in the system and represents a high heterogeneity in the network.

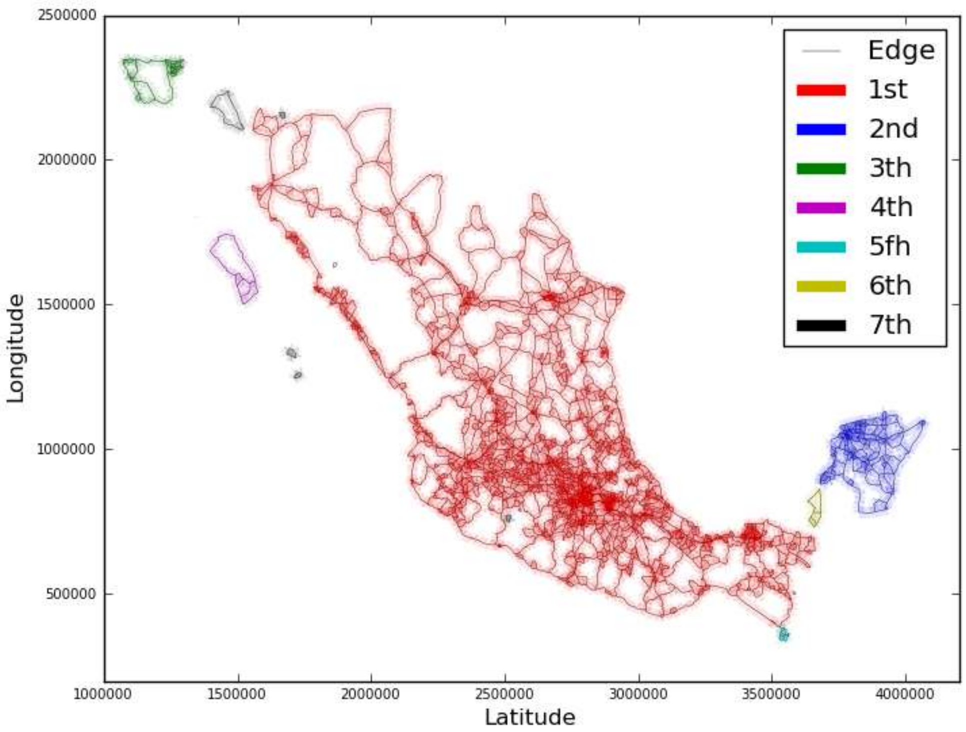
The last group of measures is divided into topological and geometrical properties of the network, where the former is related to the Cyclomatic number and α index, and the latter is correlated to circuit and tree ratios (Table 2).

The first measure in Table 2 is the Cyclomatic number, which indicates a large number of independent cycles in the network, in other words, cycles that do not contain other cycles within themselves. Additionally, α index exhibits a small value indicating a low presence of circuits in the network. Furthermore, the Φ_{ring} and Φ_{web} show a ratio of 35% and 0.08% of circuits block as rings and webs, respectively. Therefore, the value of $\Phi_{circuit}$ represents a low ratio of circuits in the network, where 35% of the system performs as circuits. In contras, the value of Φ_{tree} corresponds to a high ratio of branching structure informing that 64% of the system performs as a tree.

Table 2 - Connectivity Properties

Properties	**Value**
Cyclomatic number	2,383
α index	0.048
Φ_{ring}	0.3536
Φ_{web}	0.0008
$\Phi_{circuit}$	0.3554
Φ_{tree}	0.6455

In addition to these properties, geometric patterns related to circuits and trees structures are analyzed by geospatial maps. Figure 4 presents a visual representation of the $\Phi_{circuit}$ ratio and shows circuit structures related to hierarchical arterial roads, where the network exhibits an unconnected system of arterials, where the first, second, and third groups are related to the most important 56 national metropolitan areas in Mexico, meanwhile the rest of the arterials is related to urban areas (less than 50 thousand habitants) and rural areas [22, 17].



Source: Lugo [17]

Figure 4 - Arterials related to circuits

On the other hand, the Φ_{tree} ratio corresponds to a complete tree structure meaning that the system performs as a branching network (Figure 5).

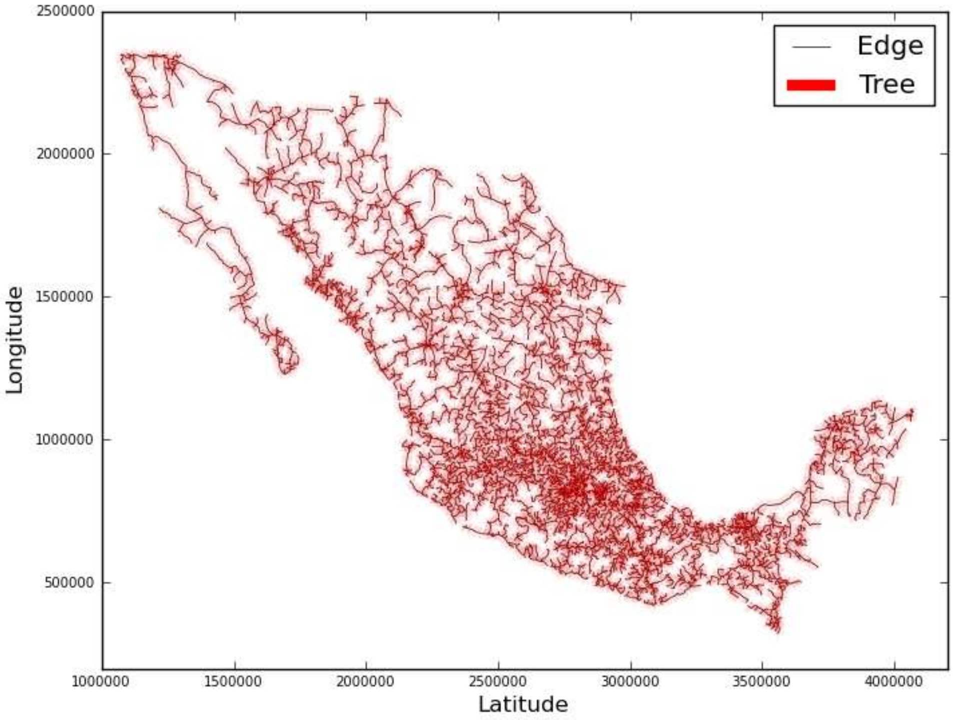


Figure 5 - Arterials related to trees

Even though this network connects all origin and destination points, it has an important restriction: there is one and only one path that connects two points. In other words, there is only one group of road segments that connects two different points. Therefore, such a restriction represents high volume and load intensity of traffic that affects the flow of vehicles and deteriorates the surface of the network.

5. DEFINING AND PRIORITIZING PAVEMENT NETWORKS AT MEXICO

According to the network definition, the complex network measures, and because we are interested in a large-scale analysis of road systems in order to determine M&R needs and priorities, the pavement network at Mexico can be defined as follow:

- Network identification. National pavements that form a complete network and connect the most important 56 metropolitan areas in Mexico.

- Branch identification. Roads related to branching and circuit structures based on a high level of mobility and connectivity between urban areas. Specifically trees and arterial roads, which include freeways and highways.

- Section identification. Segments of roads related to their level of traffic that describes their number of lanes and size that corresponds to their length.

As a result, segments of roads can be prioritized and localized on a digital map (Figure 6). The geographical information displays the most important road segments that need to be maintained and managed in order to ensure a minimum level of connectivity in the system. The map shows two priority zones related to the north and south borders of the country.

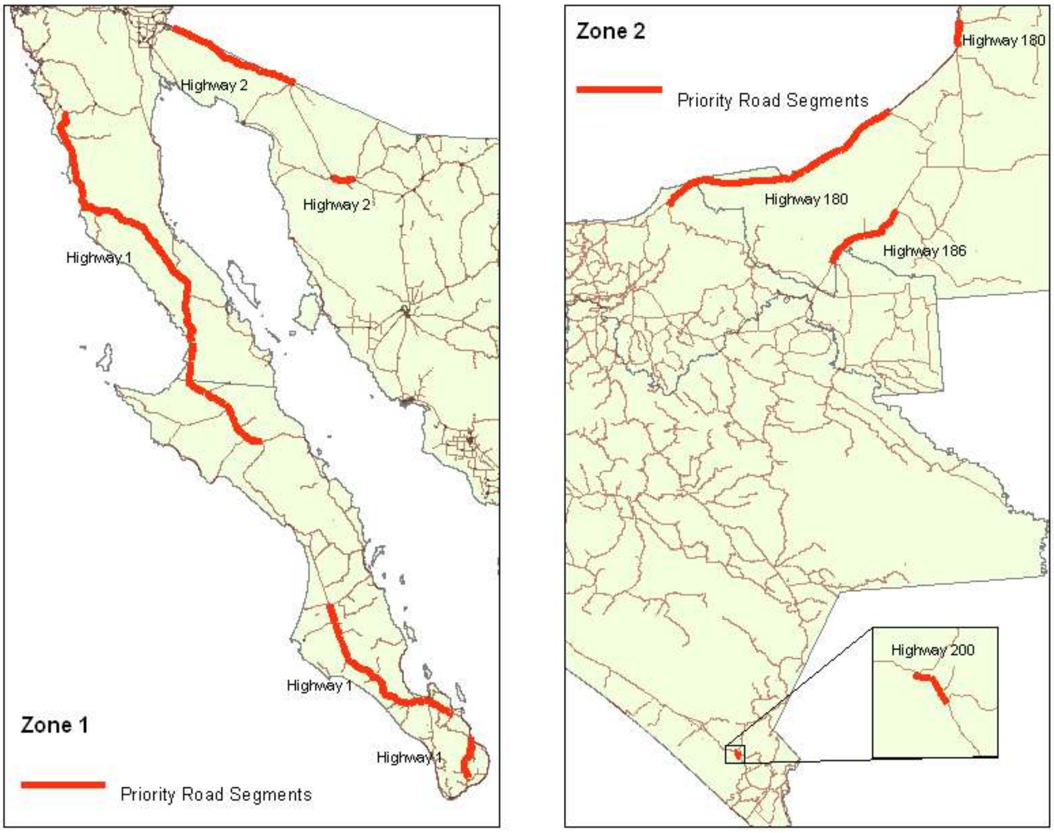


Figure 6 - Localization of Priority Road Segments

The first zone is located in Baja California, Baja California Sur, and Sonora, and it presents the following hierarchy in road segments:

1. Federal highway number 2 (Magdalena de Kino-Sanluis Rio Colorado), Sonora, from Caborca to Santa Ana (urban areas), 4 line segments, two lanes, total length equal to 33 Km.

2. Federal highway number 2 (Sanluis Rio Colorado-Magdalena de Kino), Sonora, from San Luis Rio Colorado to Sonoita (urban areas), 10 line segments, two lanes, total length equal to 196 Km.

3. Federal highway number 1 (Ensenada-Sta. Rosalia), Baja California and Baja California Sur, from San Vicente to San Ignacio (urban and rural areas), 46 line segments, two lanes, total length equal to 650 Km.

4. Federal highway number 1 (Cd. Constitución-La Paz), Baja California Sur, from Ciudad Insurgentes to La Paz (urban areas), 20 line segments, two lanes, total length equal to 277 Km.

5. Federal highway number 1 (La Paz-San Jose del Cabo), Baja California Sur, from Las Palmas to Santa Anita (urban areas), 5 line segments, two lanes, total length equal to 71 Km.

The second zone is located in Campeche, Tabasco, and Chiapas, and it presents the next hierarchy in road segments.

1. Federal highway number 180 (Champotón-Ciudad del Carmen), Campeche and Tabasco, from Ignacio Allende to Sabancuy (urban areas), 13 line segments, two lanes, total length equal to 176 Km.

2. Federal highway number 180 (Campeche-Champotón), Campeche, from Champotón to Villa Madero (urban areas), 2 line segments, two lanes, total length equal to 18 Km.

3. Federal highway number 186 (Villahermosa-Chetumal), Campeche and Tabasco, from Chablé to El Reloj (rural areas), 2 line segments, two lanes, total length equal to 61 Km.

4. Federal highway number 200 (Arriaga-Tapacula), Chiapas, from Huixtla to Huehuetán (urban and rural areas), 3 line segments, two and four lanes, total length equal to 5 Km.

To summarize, based on the connectivity importance of these roads, it is highly significant to maintain and manage them. Then, such information is used as inputs in the PMS incoming steps in order to inform and design M&R strategies.

CONCLUSIONS AND RECOMENDATIONS

The application of complex networks to the PMS adds important information to improve the pavement network definition providing a large-scale understanding of the system and focalizing road segments that are more susceptible to be maintained and managed. Therefore, complex network measures applied to the Mexican road system describe a heterogeneous structure that shows special topological attributes, low levels of connectivity, and two types of geometry patterns.

The collective measure of entropy shows a value that describes a heterogeneous structure based on a functional classification related to the number of lanes per road segment. Then, road segments explains diverse types of high level mobility, for example roads with one lane are related to local roadways that connect rural areas, and roads with more than one lane are linked with regional and national roadways that connect urban areas.

The topology of the system confirms the Mexican road network as a case of random networks. For example the network exhibits a large diameter and average path length values meaning a large number of intersections between any pair of nodes, that is, in order to move from one node to other, the distance and travel time are large reflecting a low efficiency in the system. Furthermore, low values of transitivity and clustering coefficient represent a low level of redundancy in the network. Lastly, based on the edge degree distribution, the network exhibits a limited number of connections, capacity, and length per edge.

The level of connectivity in the network shows low values. The β and γ indices indicate an underutilized road network, even though the network corresponding to one component. Moreover, the α follows the same behavior indicating a low portion of the network connected as circuits.

Therefore, there are two types of geometry patterns in Mexico: branching and circuits structures. A branching structure pervades the system while arterial roads represent a low level of participation.

Based on complex system measures, a pavement network can be defined and prioritized. Network and branch identification describe the scale and characteristics of roads to analyze. Section identification constrains the analysis to those roads with specific features. Then, a group of pavement networks can be identified and localized in order to study in detail technical aspects related to the pavement surface, for example a distress condition index and a nondestructive deflection testing.

In sum, the application of complex networks to the PMS can help private and public agencies to reduce time and avoid costly errors related to maintain and manage pavement networks. Then an effective database can be stored and analyzed for a better rational decision about the best budgeting allocation.

Recommendations for applying this kind of analysis are related to digital information and computational methods. Digital information (points and lines) need to come from reliable data source, which has to have correct geographic coordinates. In order to use this digital information and compute complex network measures, it is highly recommended to apply efficient algorithms, such as NetworkX module (Python packages), into a programming routine that reduces the computation time and improves graphical and geographical visualizations.

ACKNOWLEDGMENTS

This research was supported by ERC Grant 269826. GeoDiverCity.

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