MODELLING TOLL PLAZAS USING QUEUE THEORY

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ABSTRACT
This study aims to increase the operational knowledge of the toll plazas to maximize their performance, using the queuing theory. First, we describe the theory of queues and its appropriateness to the collection booths. We defined the concept of level of service for toll plazas with their performance standards for waiting time in queue and average queue length. It is shown a number of assumptions (input variables, criteria for the traffic division, and information output) and develops a model called Car Motion. It is verified the consistency of queuing theory to simulate the toll plaza, through calibration of the model, with error in the order of 11%. Analyses are made in five types of operational scenarios. It is proven the exponential relationship between the volume of traffic and waiting time in queue. The performance of exclusively manual booth is similar to mixed booth (manual and electronic). There is a linear relationship between increasing percentage of heavy vehicles and toll capacity reduction. The increasing of the percentage of electronic payments to 41% can reduce 33% the number of booths manuals, representing large cost savings. Finally, the use of 50% of prepaid ticket payments achieves a 38% gain in capacity.

Keywords: toll plaza; queue theory; operational scenarios.

RÉSUMÉ
Cette étude vise à accroître la connaissance opérationnelle des postes de péage afin de maximiser leurs performances, en utilisant la théorie des queues. Tout d'abord, nous décrivons la théorie des files d'attente et son adéquation aux cabines de collecte. Nous avons défini la notion de niveau de service pour les postes de péage avec leurs normes de rendement pour les temps d'attente en file d'attente et la durée moyenne de la file. Il est montré un certain nombre d'hypothèses (variables d'entrée, les critères de la division de la circulation, et la production de l'information) et développe un modèle de voiture appelé Car Motion. Il est vérifié la cohérence de la théorie de files d'attente pour simuler le péage, grâce à un étalonnage du modèle, avec l'erreur de l'ordre de 11%. Les analyses sont faites dans cinq types de scénarios opérationnels. Il est prouvé la relation exponentielle entre le volume du trafic et des temps d'attente en file d'attente. La performance de stand exclusivement manuel est similaire au stand mixte (manuel et électronique). Il existe une relation linéaire entre l'augmentation en pourcentage de véhicules lourds et la réduction de la capacité de péage. L'augmentation du pourcentage des paiements électroniques à 41% peut réduire de 33% le nombre de manuels de cabines, ce qui représente des économies importantes. Enfin, l'utilisation de 50% des paiements billet prépayé réalise un gain de 38% de la capacité.

Mots-clés: péage, la théorie de la file d'attente, les scénarios opérationnels.

1 INTRODUCTION

The roadway concession contracts require compliance with various standards. Besides pursuing these performance parameters, the operating companies also seek to minimize costs and optimize their investments. In return for payment of tolls, drivers looking for immediate results, such as roads in1 good condition and prompt services. However, many improvements made in the operation times are not perceived by users. Currently, the toll plaza becomes a key element in the operation of the business, being linked to the degree of satisfaction with the system [1].

In Brazil, the manual collection booths represent nearly 78% of total roads installed at toll plazas. This method of collection is divided basically into two perspectives. Considering the operational perspective, one can describe the high personnel costs and taxes, as well as the need for deployment of space cabins. In a user perspective, one can enumerate the discomfort expected during periods of congestion and the need to carry the cash [2]. Too many delays due to design mistakes, the design and operation of the toll plazas have a direct influence, creating a negative perception [3].

According to Michel and Senna [4], despite the benefits of electronic billing and the commitment to its implementation, this is still not widespread. Most countries now begin to implement it. Due to the high costs of mechanization and automation of the collection in the country, the resistance of the population to adhere to forms of automated billing, and legal obstacles for interoperability, studies aimed at increasing the efficiency of existing forms of recovery are still valuable.

Therefore, this study has the goal to develop an operational model of toll plaza based on queuing theory. It seeks to analyze various operational scenarios such as the influence of time of billing, service levels, number of booths, the proportion of heavy vehicles, percentage of automatic payment and prepaid payment.

2 LITERATURE REVIEW

2.1 Theory of queues

After industrial development, business growth and the need for resource optimization, the use of the methods of Queuing Theory have proved to be quite necessary [5]. The industries of transport and services are the biggest beneficiaries of these theories. However, despite the great need, there are few scientific studies applying this theory to practical situations today, such as desks of banks and supermarkets, queues, parking, entry to stadiums, and toll plazas. The queuing theory is a statistical method that allows estimating the waiting time and queue size in services provided to customers with random arrivals. Most of the time the arrival process is described by the Poisson exponential distribution, a fact confirmed at the toll plazas, for example, in the studies by Peixoto[6] and Araujo and Setti[3].

The discipline of the service determines the way users leave the queue. In the case of the operation of the toll plaza the current system is called FIFO (First In, First Out "), that is, the first in the queue is first-served and out of it [7]. Using the classification of Kendall (1953) coted at Wikipedia [7], this study will consider an M/M/1 queuing system, that is, with Markovian arrival process (Poisson exponential distribution) with time distribution of Markovian exponential service, and only one server. We used the format with one server because this is simpler in its formulation. It means that the traffic coming through the toll plaza will be equally divided on tracks, where each of these tracks serves as an independent system with only one server.

To calculate the average queuing time of each booth, we used the Equation 1 [8]. As for the average queuing time of the system, we set the average time calculated for each booth using Equation 1.

$$TF = \frac{\lambda}{\mu(\mu - \lambda)} \tag{1}$$

where: TF: average queue time [s];
λ: arrival rate [vehic/s];
μ: service rate [vehic/s].

The average queue in each booth is calculated through the Equation 2 [8]. The average queue of the system is obtained through the lenght queue average of the n booths.

$$VF = \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{2}$$

where: VF: average number of vehicles in queue [vehic];
λ: arrival rate [veíc/s];
μ: service rate [veíc/s].

2.2 Operational Service Level of a Tool Plaza

According Demarchi and Setti[9], to identify which is the volume of traffic that can move along the roadway so that a certain quality standard is maintained, was defined by the *Highway Capacity Manual –* HCM, the concept of service level. This parameter seeks to highlight the perception of users considering factors of speed, travel time, safety, comfort and more. In the case of the toll plaza, it reflects the waiting time in queue and/or the length of it. Service levels range from A to F, where the standard refers to the best traffic conditions, and level F the profile of saturation or congestion. The level E corresponds to the operational capacity.

Several authors have defined levels of service for toll plazas such as Faria[9], & Al-Deek Klodzinski (2002) apud Campos and Faria[10] and Araujo & Setti[1]. However, this study will use a scale of service level based on the work of Hokao & Somasundaraswaran[11] held in Bangkok, Thailand. The authors used this scale as a basis, because it fits better in the empirical standards and operating toll plazas of their study (see Table 1). In following tests carried out in this work, the service levels will be demonstrated in the graphics to improve understanding of the conclusions.

Table 1 – Level of Service for Squares Toll.
(adaptedfromHokao&Somasundaraswaran, 1995).

Level of Service	Time in Queue	Average Queue (vehic)
A	<32	<1
B	<=54	<=2
C	<=76	<=3
D	<=135	<=6
E	<=271	<=15
F	>271	>=15

3 DEVELOPMENT OF THE MODEL

After the knowledge of queuing theory, it was possible to set the systematic model of operation of the toll plaza in a Microsoft Excel spreadsheet data, named "*Car Motion*". In preparing the same, some assumptions have been complied in order to streamline and simplify its formulation without losing scientific rigor:

- The input variables are:
  - 1) Traffic flow time (vehic/h);
  - 2) Number of booths per type of operation (electronic, manual and mixed);
  - 3) Time of service for each vehicle type and form of payment. These times were collected in the field, consisting of headway since the vehicle arrest in the booth to its passing through the sensor that activates the gate (see Figure 1). Most studies consider the time between stopping the vehicle to lower the gate. However, it is known that the billing systems allow receiving the fee when the vehicle passes the sensor precedent, even before bringing the gate down.

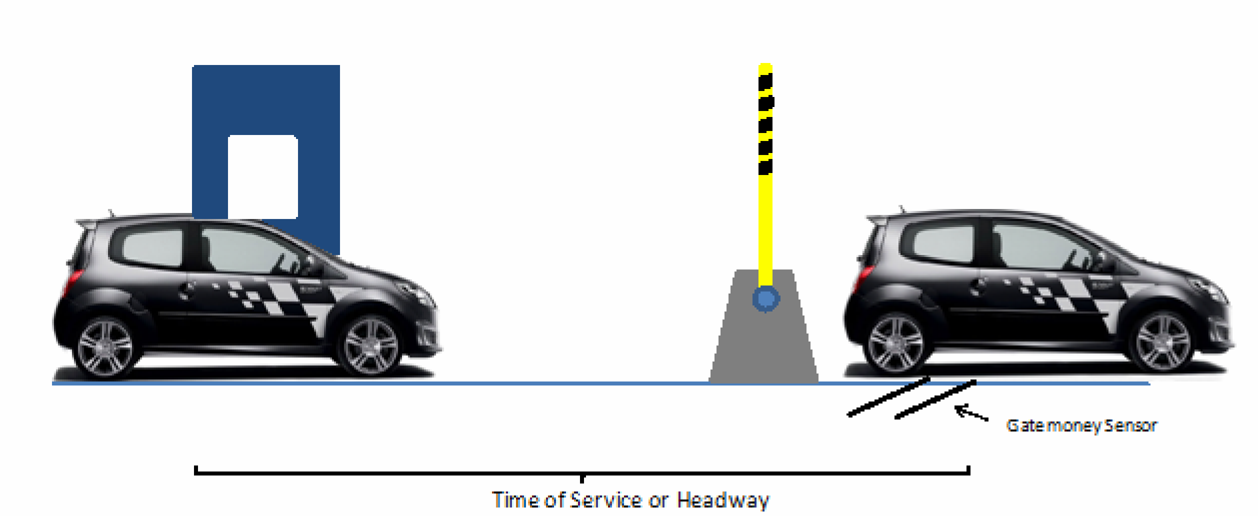

Figure 1–Headway between two vehicles (time of Service).

  - 4) Fleet Composition (percentage amount of cars, trucks and buses).
  - 5) Percentage Distribution of Traffic by Way of Payment. The payment forms adopted are AVI (electronic payment); users granted by the concessionaire (they use contactless card); Signup Coupon; Signup to Smart Card, Cash

(no distinction between payment with or without change) and exempt users.

- The criteria of division for incoming traffic in the toll is the following:
  - 1) Traffic with electronic payment (AVI) is distributed equally in automatic and mixed tracks;
  - 2) traffic with manual payment is divided equally between manual and mixed tracks;

    It is known in practice that sometimes the flow of vehicles in the toll can create some confusion causing overburdened roads and idle roads [12]; however, this fact will not be considered in this model.

- The information of output of the model:
  - 1) Average Number of vehicles in queue, calculated from the average queue in each booth type weighted by the volume of traffic on each of them;
  - 2) Average time in queue, average time of each booth type multiplied by the proportion of traffic in each one.

3.1 Calibrating of the Model

In order to validate the model, it should be compared to real data of operation of the toll plaza. The calibration process was conducted in two phases: the first was held on January 29, 2010, in the toll plaza itself, where 90 observations about time of wait in the queue were made and about queue size. The second collection of calibration data was held on 31 January 2010, within the Operations Control Center at company headquarters. This second collection was made through cameras, where 43 measurements were performed queue size. In the second test, the waiting time was not measured. In both measurements, the input parameters of the model were equal to those measured in hours of actual field.

Table 2 summarizes the data measured in field and by the model. The first column of measurement data field is placed after the number of manual, mixed and electronic booths in operation. We describe the time interval measurement, the direction (ascending or descending) and the daily traffic in this direction. It is also expressed the average time in queue and average queue of vehicles measured in the field, and the same data obtained by the model. We calculated the percentage difference between the model and reality, and we made an average modulus of these gaps. Therefore, it is possible to verify the validity of the model, since it hits with an error of 11.3% the waiting time and 33.6% the size of the average queue of vehicles in the queue. These percentages of correct answers are pretty good for models with this purpose.

Table 2 – Calibrating the Model.

Date	Pathways			Hour	Direction	Traffic	MEASURED		MODEL		DIFFERENCE %		MODULE	
	Man	Mix	Elet				Time in Queue	Average Queue	Time in Queue	Average Queue	Time in Queue	Average Queue	Time in Queue	Average Queue
29/01/2010	2		1	09:00:00	D	250	00:00:28	0,65	00:00:33	1,04	-15,9%	-37,5%	16%	38%
29/01/2010	3		1	09:00:00	A	284	00:00:21	0,64	00:00:18	0,41	14,6%	56,2%	15%	56%
29/01/2010	2		1	10:00:00	D	254	00:00:36	0,64	00:00:35	1,1	2,4%	-42,2%	2%	42%
29/01/2010	3		1	10:00:00	A	292	00:00:19	0,43	00:00:18	0,45	2,9%	-3,5%	3%	4%
29/01/2010	2		1	11:00:00	D	241	00:00:24	0,65	00:00:30	0,9	-21,0%	-27,8%	21%	28%
31/01/2010	3	1		17:00:00	D	628		4,15		4,03		3,0%		3%
31/01/2010	2		1	17:00:00	A	209		0,78		0,56		39,3%		39%
31/01/2010	2		1	18:00:00	A	248		1,75		1,01		73,3%		73%
31/01/2010	2		1	19:00:00	A	206		0,63		0,53		19,4%		19%
												Average	11,3%	33,6%

4 EVALUATION OF RESULTS

There are many types of analysis that can be developed with the *Car Motion* model. We selected the five evaluations considered the most important for discussion and optimizing the operation of toll plazas, which are described below.

4.1 Average time in queue by day time

The first analysis performed was the average waiting time in queue at each time of day. To do so it is considered a toll plaza booths operating with three manuals and one mixed booth with 8.3% of users making payment AVI (electronic). This scenario is exactly the profile of an average operation of the senses of the toll plaza used as parameter for assembling and calibrating the model. The capacity of this effect (level of service E) is equal to 701 vehicles, with a waiting time of 271 seconds and average queue of 15 vehicles.

It is observed in Figure 2 that the traffic sense of the toll plaza begins to increase gradually from 6 am reaching the peak at 7 pm. The average waiting time in queue roughly follows a behavior directly proportional to traffic. However, while at 4m it goes up to 47% capacity (333 vehicles) to 68% capacity (481 vehicles) at 6 pm (an increase of 21% utilization of capacity or 44% traffic) queue timing is more than the double, going from 14.7 s to 32.7 s. This fact is due to the exponential behavior of the waiting time in queue. There is a certain point of occupation where a small capacity increase traffic represents a large increase in waiting time.

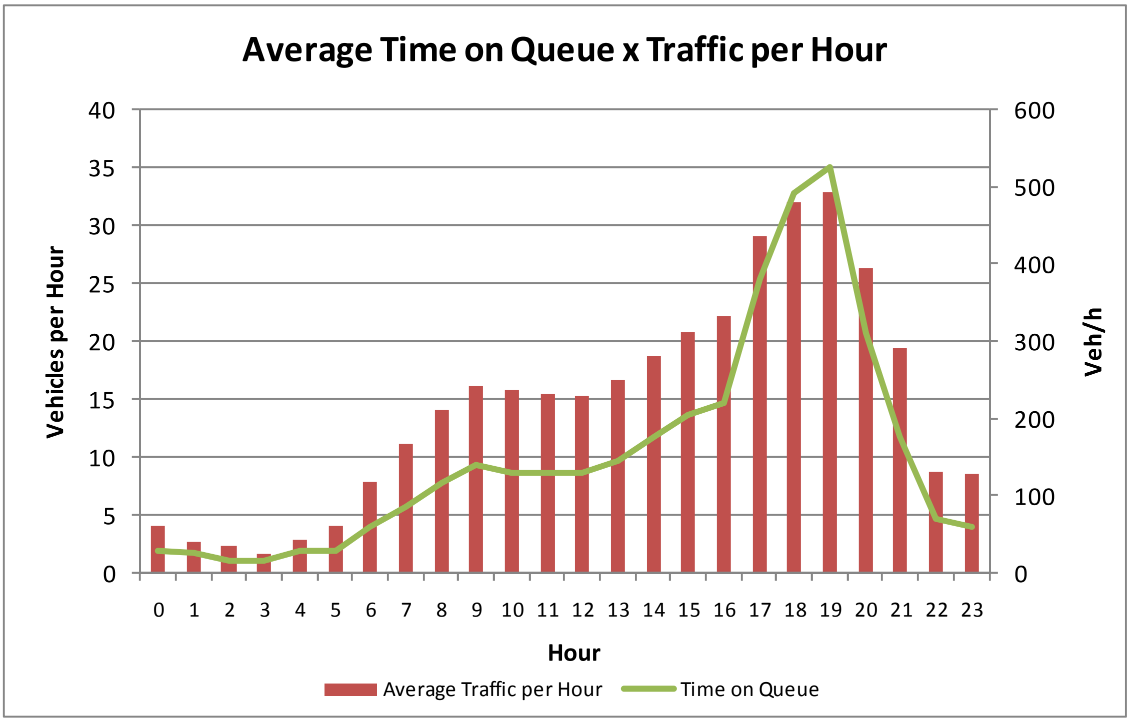

Figure 2 – Average Time on Queue per hour and per day.

4.1 Influence of the number of booths and of the type of operation in capacity

In Figure 3, we analyze the relationship between the number of booths in operation, the volume of vehicles per hour and the average waiting time in queue for 8.3% of electronic payment (standard operating current). First, we observe that when we increase a manual or mixed cabin at the toll plaza, the ability of the toll (and service level) is increased by 180 vehicles per hour. Through the analysis of the graphic, we see that the mixed cabins have a behavior virtually identical to the manuals ones in a lower percentage of electronic payment.

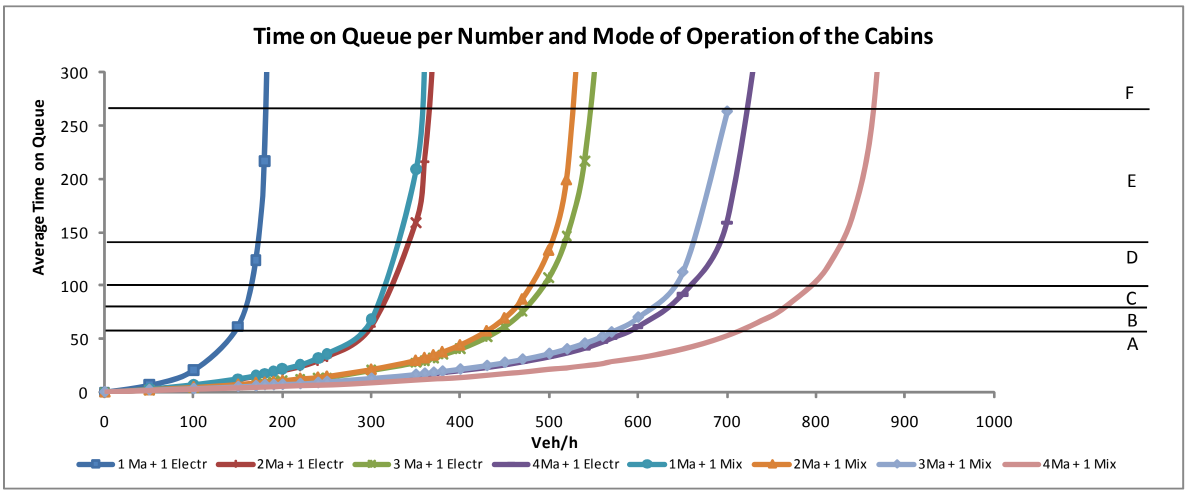

Figure 3 – Average Time on Queue per Number and Mode of the Cabins.

4.3 Influence of commercial traffic in the capacity

Below we will analyze the relationship between the share of commercial vehicles (trucks) in operating capacity, considering an example with 3 manual cabins and 1 mixed cabin using 8.3% of electronic payment (common situation of the toll plaza prototype). It appears that every 10% more commercial vehicles in the operation reduceabout 5% of the total processing capacity (see Figure 4).

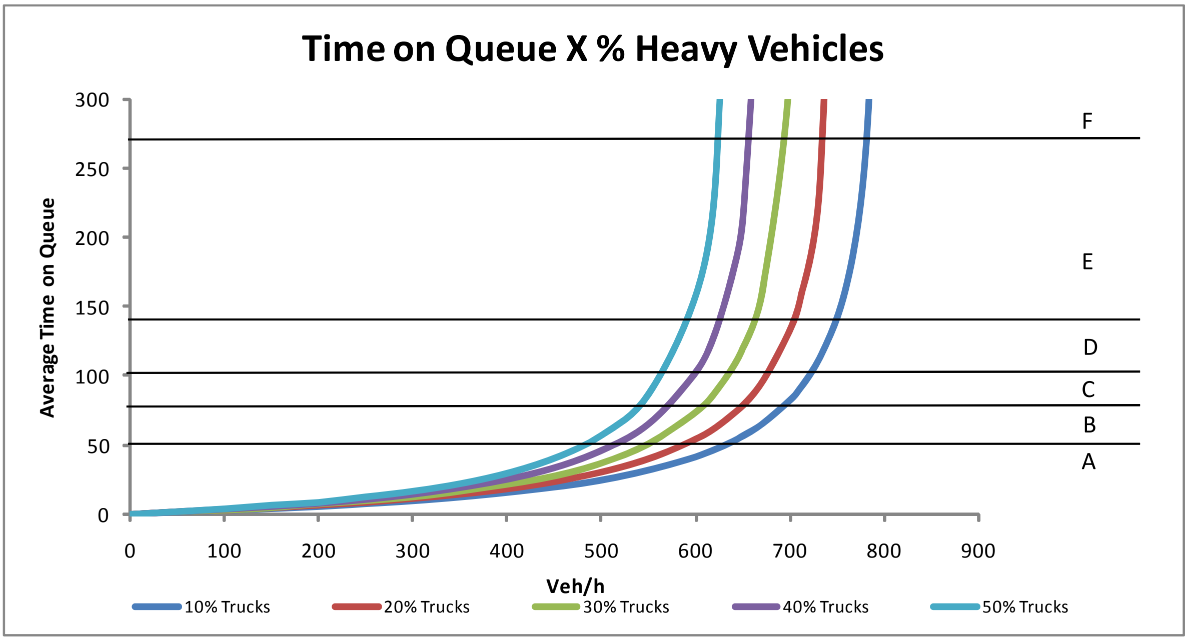

Figura 4: Influence of Percentage of heavy vehicles on the capacity.

4.4 Influence of percentage of electronic payment on the capacity

One of the most discussed ways to increase the operational capacity of the toll plazas and to improve the comfort of the drivers is the increasing proportion of electronic payments [13]. However, there are few studies that measure the real gains of these devices. So below there an example is developed in order to describe these measurements.

Considering an example with three manual cabins and one electronic cabin in operation, the Figure 5 shows an important relationship between the percentage of payment capacity of the plaza and AVI (operational level E). Is is possible to see that when there is an increase of the percentage of AVI payment to 20%, there is a 7.9% increase in the capacity of the square.

Pushing up the share of electronic payments to 30%, there is a gain of 21% of the capacity. With a percentage of 40% from AVI, there is an advantage of 44% in the capacity. Finally, with an insertion of 50% of electronic payment, the capacity increases 75%.

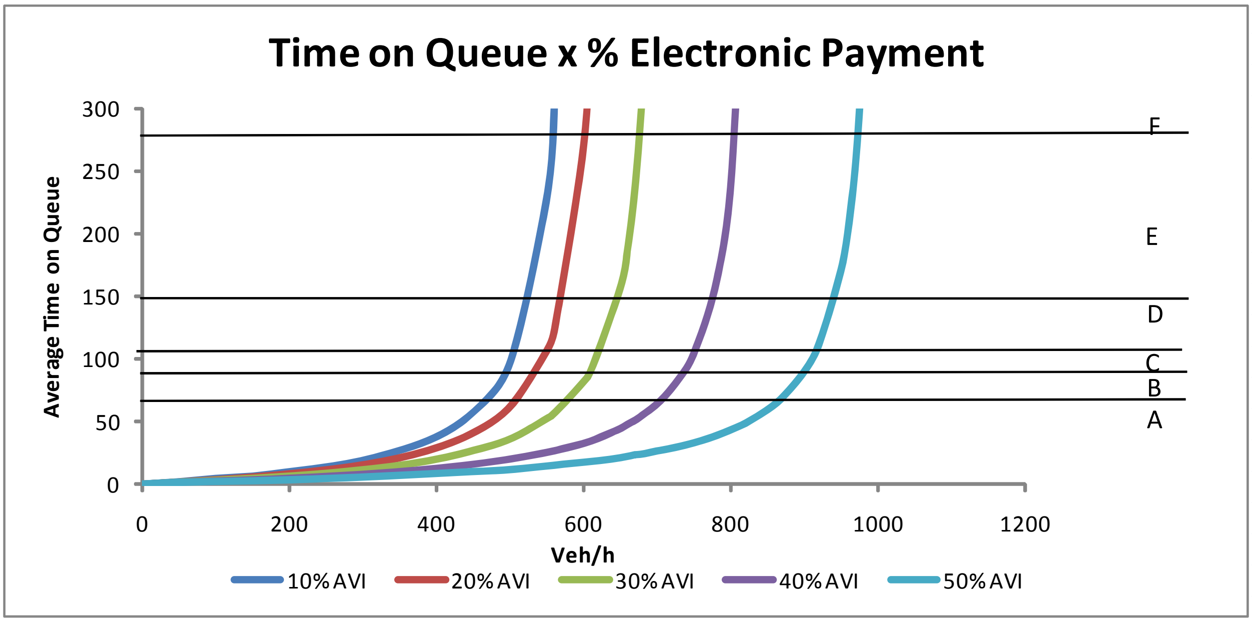

Figure 5 – Influence of percentage of electronic payment on the capacity

For the company operating the toll, it is important to know what the target percentage of electronic payment inclusion is in order to eliminate manual cabins (see Figure 6).This fact reduces operating costs and increases efficiency. So in the example with three electronic cabins and one manual cabin in operation and 41% of automatic payment, it is possible to eliminate one manual cabin. In other words, this means that with operational mix with 41% AVI payment is possible to remove 33% of the manual cabins.

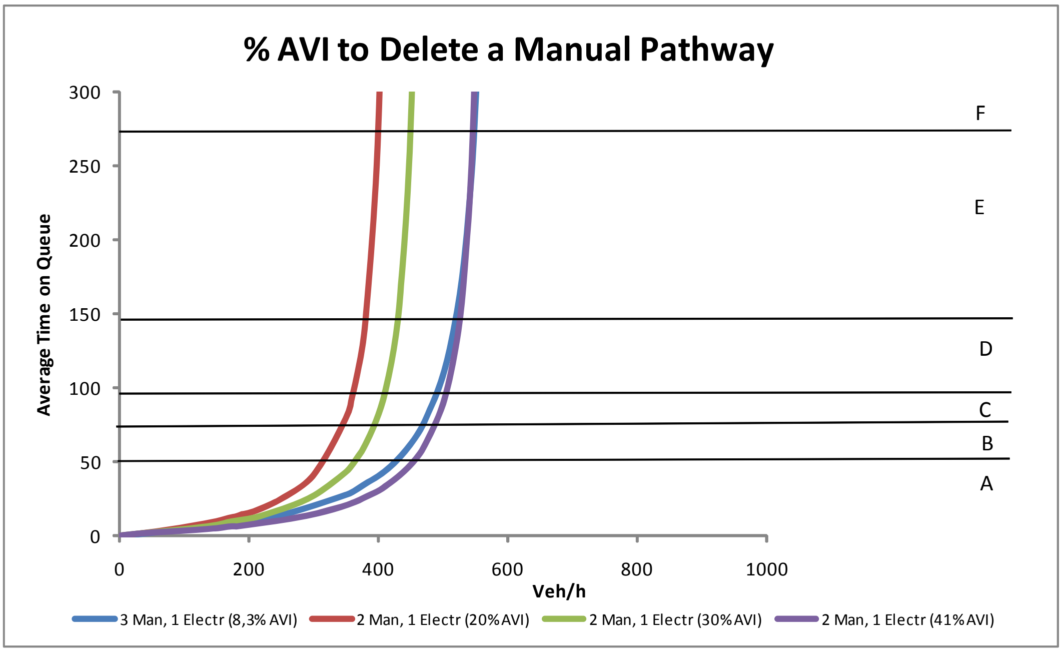

Figure 6 – Percentage of electronic payment to eliminate manual cabins.

4.5 Influence of prepaid ticket payment

One way often used to increase the capacity of the toll plaza at atypical moments and holidays is by using pre-paid tickets in which the payment is made by the user while waiting in queue. In the cabin, the user just gives the ticket, which speeds the payment, with an average time of service of 12 seconds instead of one of 21 seconds (with manual type). In the Figure 7, it is noted that an insertion of 10% of payment made with repaid tickets increases 6% the capacity of the toll plaza. A use of 30% of this type of payment incrementsto 20% the capacity of the toll. Finally, a use of 50% of prepaid tickets means a 38% gain in capacity, as 60% of pre-paid coupon payment means an increase of 49% of the capacity.

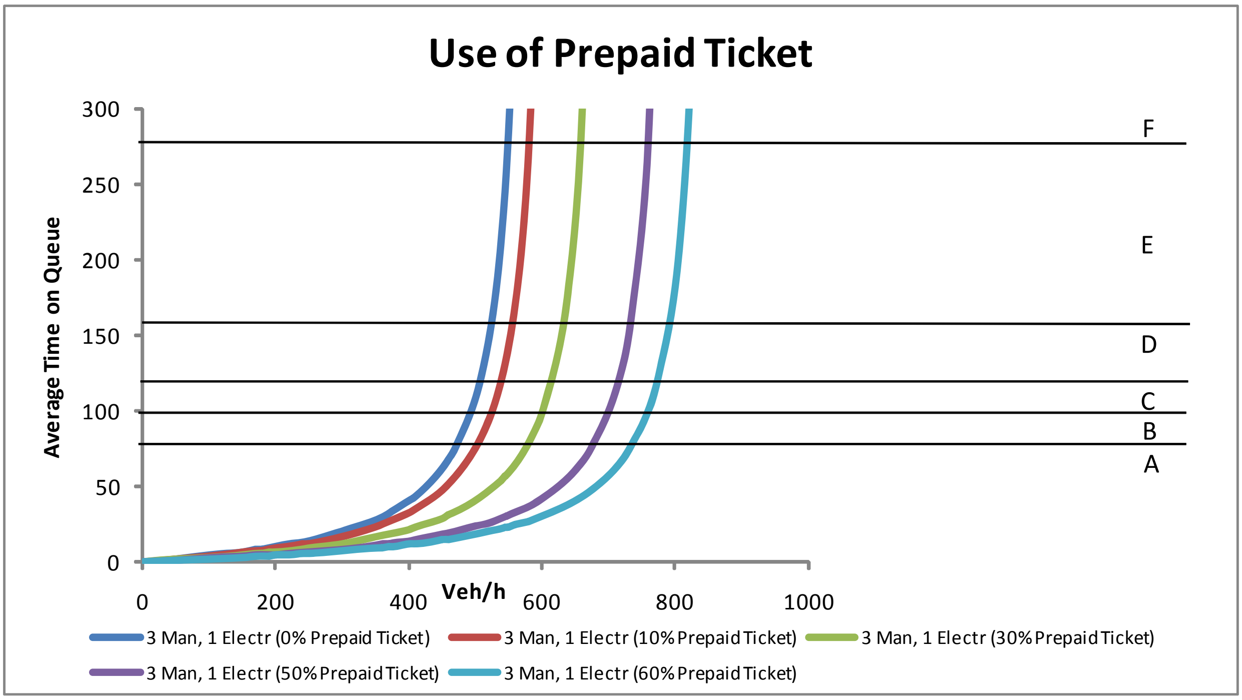

Figure 7 – Influence on Payment with prepaid ticket over the capacity of the toll.

5 FINAL REMARKS

After the development of this work, we concluded that the formulation of the theory of queues with Marjovarrival process and service with only one server (M/M/1) was suited for simulation of the toll plazas. This fact was confirmed by calibrating the model with real field data, achieving consistent values of accuracy: 11% error for time waiting in queue, and in 33% of divergence for the number of vehicles in the queue. Thus, this type of formulation is recommended due to its simplicity compared to other existing methods. The main results were as follows:

- In relation to the average time in queue per time of the day, there was an exponential behavior of the volume of traffic and waiting time. For small values of traffic, the relationship is almost linear. However, there is a certain amount of capacity occupancy (above 35% capacity) where small increments represent large increases in traffic waiting time;
- Regarding the influence of the amount of booths and the type of operation, it was found that mixed cabins are similar in behavior to the low percentages of manuals for electronic payment. The inclusion of a cabin of any of these types increases in capacity 180 vehicles;
- In relation to the impact of commercial traffic, it was found that every 10% more trucks reduces to about 5% the capacity of the toll plaza;
- Regarding the percentage of electronic payments, it is observed that when the percentage is increased from 8.3% to 30%, there is a gain of 21% in the capacity of

the toll plaza. And with an increase to 50% of electronic payment, there will be an extra capacity of 75%;

- It is necessary to arrive at a percentage of 41% of electronic payment to remove 33% of the existing manual booths;
- The use of prepaid vouchers was fairly valued after the analysis. It was found that when 50% of sale is made through prepaid coupon, there is a gain of 38% in operating capacity of the toll plaza.

It is advisable to compare the values proposed by the analysis of this study with the future real values. Furthermore, we suggest the use of the *Car Motion* model for toll plazas of larger capacity in order to validate it in double lane highways and freeways. It is also interesting to compare the effectiveness of this model in relation to other simulation models and computer graphics.

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