PROBABILISTIC DESIGN PROCESS FOR ASPHALT PAVEMENTS

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ABSTRACT RÉSUMÉ

The design process of asphalt pavements according to the German guideline RDO Asphalt 09 [1] based on a quasi-semi-probabilistic method. The traffic loading and temperature conditions (both are relevant input parameters for the design process) will be considered as probabilistic parameters. These parameters will be used within the computational design process in terms of empirical distributions. Other relevant input parameters for the design process in particular all the material properties and layer thicknesses will be considered as deterministic input parameters. For the improvement of computational accuracy concerning design of new asphalt structures and calculations the residual structural life of asphalt structures can be realized by using the purely probabilistic method. Furthermore it is possible to determine the probability of failure of the asphalt structure. This is an important innovation in the design process of asphalt pavements.

1. INDRODUCTION

A significant step towards a more reliable and economically as well as environmentally sustainable design of asphalt pavements has been made with the German guideline for the design process of asphalt pavements (RDO Asphalt 09) [1]. The underlying methodology in the RDO Asphalt 09 [1] enables to consider significant input parameters which influence the durability of pavement structures. One of the most important parameters is the traffic loading. These will be considered in the design process by axle load distributions. Thus, stresses and strains depending on the individual axle loads can be calculated and estimated within the framework of necessary proofs.

Within the design process of asphalt pavements the relevant material properties will be considered by the material-specific stiffness-temperature functions and the fatigue functions. The relationship between asphalt temperature and asphalt stiffness is realized by special temperature profiles. A procedure for individual consideration various parameters on the stresses and strains of asphalt pavements is provided to the road construction engineer with the asphalt RDO 09 [1]. Thus, the thickness of the construction layers can be designed accordingly. However, each user must be known that the precision of the calculation result depends on many different factors. These include:

- the appropriated method of calculating
- the appropriated rheological models for the road construction materials
- the quality of laboratory experiments to capture and describe the material parameters for the rheological models
- the quality of description of load conditions including their forecast

These uncertain factors will be considered by a safety factor and a shift factor. While the safety factor collected all non exact loading conditions (for instance: load breaks, influence of loading rate, load function, etc.), the shift factor considered the imprecise assumptions of the material properties (for instance: stiffness-temperature function, fatigue function, thickness of structure layer, bearing capacity, etc.) as well as the imprecise results in consequence of simplified model assumptions and calculation methods.

The current quasi semi-probabilistic procedure has to be transferred in a purely probabilistic procedure to consider the scatters of the material properties in the design process adequately. Therefore, the input parameters and material properties will be considered as random variables and described exemplary by probability density functions and distribution functions respectively. Moreover a procedure for classification of these random variables has to be defined. This ensures that always the same distribution results follow from the same sample. With the help of the probabilistic procedure, the probabilities of failure of asphalt structures can also be calculated. The Calculation of probability of default is an important innovation in the design process of asphalt pavements. Therefore, the user will be provided a tool that allows considering individual need of security.

2. STATUS QUO OF THE RDO ASPHALT 09

The current version of the design process according to RDO Asphalt 09 [1] can be described as a quasi semi-probabilistic procedure (Figure 1). The computing process is based on empirical distributions concerning the input parameters traffic loading and temperature conditions.

The entire spectrum of traffic loads is described by a statistical distribution of 11 axle load classes (from 2 t until 22 t with a class range of 2 t) [2]. The reference value of every axle load class is defined by the upper class limit. Altogether, three different axle load distributions will be considered currently.

The temperature conditions that could occur will be considered by temperature profiles according the RDO Asphalt 09 [1]. These temperature profiles are depending on the surface temperature. The full range of surface temperatures has been classified into 13 temperature classes with a constant class range of 5 K [3]. The reference value of every temperature class is the average temperature. The probability of occurrence is defined by a statistical distribution of the 13 temperature classes. Furthermore, the different German climatic conditions will be considered using different surface temperature distributions and a pavement temperature map respectively.



Figure 1 - Schematic illustration of the procedure of the design process of asphalt pavements [3].

The input parameters axle load and temperature conditions are discrete random variables and can be represented as set of special value of random variable in following form:

$$V = \{v_1, v_2, ..., v_{11}\}$$

$$T = \{t_1, t_2, ..., t_{13}\}$$
(1)
(2)

Where T, V = random variable (set of all temperature conditions, of all axle loads); t_i , v_j = special value of random variable (temperature state with i=1...13, axle load state with j=1...11).

The number of all necessary realisations of the combined random variables (load conditions) results from the Cartesian product of set V and T.

$$B = V \times T = \{b_1, b_2, ..., b_{143}\}$$
(3)

Where B = combined random variables (set of all load conditions); b_a = special value of combined random variables (load condition with a=1...143).

Altogether 143 values (load conditions) will be considered according to the RDO Asphalt 09 [1]. The input parameters axle loading and temperature conditions will be collected and considered using empirically determined distributions according to the current version of the RDO Asphalt 09 [1]. Other input parameters, for example the material properties or the thickness of the pavement layers and full-depth asphalt pavement respectively will be described by deterministic values.

The stiffness-temperature function that is approximated from laboratory test results will be used for calculating stiffness profiles in the pavement structure [4]. In a similar procedure the fatigue function will be determined by approximation the test results. Both functions are only mean value functions of the appropriate underlying samples [4].

The scatters / variability of the material properties and their influence on the results of the design process will be unaccounted in the current procedure. The effects of this scatters / variability will be considered using a safety factor. The thickness of full-depth asphalt construction will be also defined as a deterministic value. In contrast to the stiffness modulus and the fatigue properties the absolute minimum (instead of the average value) of thickness of full-depth asphalt construction will be used in the current design process.

According to the RDO Asphalt 09 [1] the inbuilt thickness of full-depth asphalt construction shall not be less than the calculated thickness. The deformation parameters of the unbound and hydraulically bounded materials that are necessary for the design procedure will be used as deterministic values in the current version of the RDO Asphalt 09 [1]. The scatters of these parameters will also be neglected and their influences will be considered by the safety factor.

For the set of all load conditions B with $B = b_1 \dots b_{143}$ the stresses and strains will be calculated in all authoritative proof points using the multi-layer theory. Following the necessary proofs will be conducted. At this, evidence is presented for sufficient thickness of full-depth asphalt construction. The necessary proofs are:

- Fatigue proof of asphalt
- Fatigue proof of hydraulically bound subbases
- Deformation proof of unbound subbases and formation

3. PROBABILISTIC PROCEDURE

3.1. Principles

The relevant parameters that are defined as random variables essentially are shortly described in section 1 and 2. The random variables have to be classified as parameters that are variable relating to an object (object variable) on the one hand and as variables that are constant relating to an object (object constant) on the other hand. Object constant variables are random variables that are local changeless relating to a pavement structure with the length L. This includes for example the temperature profiles, the surface temperatures as well as the axle loads. In contrast, object variable random variables are parameters that are not restricted to any particular variables referring to a pavement structure with the length L. This includes for example the thickness of pavement layers and of full-depth asphalt construction respectively, the stiffness modulus of asphalts and the fatique behaviour (allowed number of load cycles referring to a defined elastic stain). The variability of these state variables, which are defined as random variables, results from inhomogeneity of the pavement materials. Furthermore, the scatters of these state variables can be caused by paving technology. In any case, this variability should be taken into account in the pavement design process. The distinction between object variable variables and object constant variables is necessary for calculating probability failure.

Describing random variables by probability density functions and distribution function respectively are the basic principle to use the probabilistic procedure in the course of the pavement design process. The probability density functions and distribution functions respectively can be directly determined empirically from the sample when the sample size is large. An empirical determined probability density functions and distribution function respectively can be caused large inaccuracies of the model using small sample size. In this case, a stochastically modelling of the random variables using theoretically probability density functions and distribution functions.

3.1.1. Classification of random variables

The pavement design process according to the RDO Asphalt 09 [1] based on a numerical algorithm. Thus all input variables (parameters and boundary conditions) are precise numerical values.

The state variables (thicknesses of pavement layers and of full-depth asphalt construction respectively, stiffness modulus, fatigue behaviour, bearing capacities) that are defined as random variables are distributed continuously. They can assume any value into a defined and at least one-side limited interval. Due to the numerical algorithm the continuous random variables have to be transformed into discrete variables. This is conducted by the classification of the random variables. The model error resulting from the classification depends on the number and size of the chosen classes. The smaller the class size and the larger the class number, the lower the resulting model error.





Concerning the classification of continuous random and state variables, laws have not been considered as a matter of principle. This means that class size and class number can be selected arbitrarily and the size of every defined class can be determined variably. The class number including the size of every class will be defines depending on their influence to the results of the design process – this is a result-oriented classification. State variables that have a minor impact on the results of the design process can be discretised using fewer and wider classes. Even if small variation of state variables have large result variation the class size and class numbers have to be discretised as small and as many as possible.



Figure 3 - Schematic illustration of classification of a continuous random variable using different class numbers but constant size of every class (upper figure) as soon as different class numbers and variable size of every class (lower figure).



Figure 4 - Schematic representation of the result-oriented classification.

The value set Ξ of the continuous random variable X will be transformed into a set Ξ of discret random variables x_i by the classification process.

$$\Xi = \{X \mid x \in R\} = \{x_i \mid 1 \le i \le k\} \text{ mit } x_i \in R, k \in \mathbb{N}$$
(4)

Discret random variables have discrete distribution functions and every value has a probability. The probability for the accidental occurrence, that X is into the intervall x_i until x_{i+1} , can be calculated by integration of the theoretical probability density function using for the stochastical modelling of the random variable from the integration limits x_i to x_{i+1} .

$$P(X \in [x_i, x_{i+1}]) = \int_{x_i}^{x_{i+1}} f(x) dx$$
(5)

3.1.2. Stochastic independency

Stochastic independency is a special probabilistic concept. This means that random events do not influence each other. If the probability of the random variable X for any open interval is independent of the values y of the random variable Y, than both random variables are stochastic independently. In this case the probability of the simultaneous occurrence of two events is defined by:

$$P(X \in [x_i - \frac{\Delta x_i}{2}, x_i + \frac{\Delta x_i}{2}] \cap Y \in [y_i - \frac{\Delta y_i}{2}, y_i + \frac{\Delta y_i}{2}])$$

$$= P(X \in [x_i - \frac{\Delta x_i}{2}, x_i + \frac{\Delta x_i}{2}]) \cdot P(Y \in [y_i - \frac{\Delta y_i}{2}, y_i + \frac{\Delta y_i}{2}]) = P(x_i, y_i)$$
(6)

For all random variables that will be considered in the probabilistically pavement design process the stochastic independency will be a priori defined. Contrary evidence are not available.

3.2. Principle of procedure

The difference between object constant and object variable random variables were explained in the previous chapter. The object variable random variables include only the material properties as soon as the thicknesses of pavement structure. These are classified into characteristics and comparison parameters.

The characteristics includes all material properties and thicknesses that are necessary to calculate stresses and strains into the pavement structure directly (thicknesses of pavement layers and thickness of full-depth asphalt pavement respectively, stiffness modulus of used asphalt, bearing capacity of unbound materials). However, the comparison parameters (fatigue behaviours) will be exclusively needed for proofs.

Figure 5 shows the principle of the probabilistically pavement design procedure schematically. The procedure consists of 6 different modules that are described briefly below.



Figure 5 - Schematic representation of the probabilitically pavement desing procedure.

3.2.1. Module 1: Load variables - realisations and there combinations

The load variables include the traffic loading considered by axle loads as well as the temperature conditions represented by temperature profiles in combination with surface temperatures. A set of finite states of load variables per load variables (set of special values) will be obtained classifying all axle loads and surface temperatures whatsoever.

$$V = \{v_i | 1 \le i \le k_v\}$$

$$T = \{t_i | 1 \le j \le k_T\}$$
(8)

Where T, V = set of all load variables (temperature conditions and axle loads); t_i , v_j = special value of the load variable (special axle load and special temperature condition); k_V , k_T = number of values = number of classes (axle load classes and temperature classes).

Every realisation of the load variables describes a class interval:

$$v_i \in [v_i - \frac{\Delta v_i}{2}, v_i + \frac{\Delta v_i}{2}]; t_j \in [t_j - \frac{\Delta t_j}{2}, t_j + \frac{\Delta t_j}{2}]$$

The axle loads as well as the temperature condition are random variables and their individual probability can be assigned by:

$$P(V \in [v_i - \frac{\Delta v_i}{2}, v_i + \frac{\Delta v_i}{2}]) = P(v_i) \text{ with } \sum_i P(v_i) = 1$$
 (9)

$$P(T \in [t_j - \frac{\Delta t_j}{2}, t_j + \frac{\Delta t_j}{2}]) = P(t_j) \text{ with } \sum_j P(t_j) = 1$$
 (10)

Both load variables are independently in terms of stochastic. The probability of common occurrence of a special value of axle loads v_i and a special value of the temperature conditions t_i can be calculated by the product of both single probabilities.

$$P(v_i, t_j) = P(v_i) \cdot P(t_j) = P(b_a)$$
(11)

The set of all combined load variables B resulting from the sets of load variables V and T and can be determined by the Cartesian product.

$$B = V \times T = \{(v_i, t_j) \mid v_i \in V, t_j \in T\} = \{b_a \mid 1 \le a \le k_B\} \text{ mit } k_B = k_V \cdot k_T$$
(12)

Where B = set of combined load variables; b_a = special value of the combined load variables; k_B = number of values = number of combined classes.

3.2.2. Module 2: characteristics - realisations and there combinations

The stiffness of asphalt, the thickness of full-depth asphalt pavement and the bearing capacities of unbound material among other things are necessary for calculating stresses and strains in the pavement structure. For these characteristics only one realization (the average value and the minimum respectively) has been considered in the previous semiprobabilistic pavement design process. In a purely probabilistic process these characteristics have to be described and used as random variables. For the set of classified characteristics (= set of all special values), it is:

$$\begin{split} D &= \{ d_1 \mid l \leq l \leq k_D \} \\ E &= \{ e_u \mid l \leq u \leq k_E \} \\ F &= \{ f_g \mid l \leq g \leq k_F \} \end{split}$$
 (13)
(14) (15)

Where D, E, F = set of characteristics (thickness of full-depth asphalt pavement, stiffness module, bearing capacity); d_I , e_u , f_g = special values of the characteristics; k_D , k_E , k_F = number of values = number of classes.

Every realisation of the characteristics describes a class interval equivalent to the load variables:

$$d_{1} \in [d_{1} - \frac{\Delta d_{1}}{2}, d_{1} + \frac{\Delta d_{1}}{2}] ; e_{u} \in [e_{u} - \frac{\Delta e_{u}}{2}, e_{u} + \frac{\Delta e_{u}}{2}] ; f_{g} \in [f_{g} - \frac{\Delta f_{g}}{2}, f_{g} + \frac{\Delta f_{g}}{2}]$$

With the probability density functions that are assigned to continuous realisations of characteristics the probability of every discrete realisation can be calculated. The three characteristics that are exemplary described in this article can be defined as stochastically independency.

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The probability of common occurrence of special realizations of the thickness of full-depth asphalt pavement d_l , the stiffness module e_u as well as the bearing capacity f_g can be calculated by the product of all three single probabilities.

$$P(d_1, e_u, f_g) = P(r_h) = P(d_1) \cdot P(e_u) \cdot P(f_g)$$
(16)

The set of all combined realisations of the characteristics R can be estimated by the Cartesian product again.

$$R = D \times E \times F = \{(d_1, e_u, f_g) \mid d_1 \in D, e_u \in E, f_g \in F\} = \{r_h \mid 1 \le h \le k_R\} \text{ mit } k_R = k_D \cdot k_E \cdot k_F$$
 (17)

Where R = set of combined characteristics; r_h = special value of the combined characteristics; k_R = number of values = number of combined classes.

3.2.3. Module 3: stress and strain conditions

According to the previous semi-probabilistic design process stresses and strains will be calculated for all realisations of combined load variables b_a in the purely probabilistic design process again. The proofs must be conducted for one special realisation of combined characteristics (average and minimum values respectively) using the semi-probabilistic design process. However stresses and strains have to be calculated for all realisations of combined characteristics r_h using the purely probabilistic design process. An especially realisations of stresses and strains conditions β_z results from an especially realisations of combined load variables b_a as well as especially realisations of combined process and strains conditions \dot{B} can be calculated by the product of B (set of combined load variables) and R (set of combined characteristics):

$$\dot{B} = B \times R = \{\beta_z \mid 1 \le z \le k_{\dot{B}}\} \text{ with } \beta(b_a, r_b) = \beta_z \text{ and } k_B = k_B \cdot k_R = k_V \cdot k_T \cdot k_D \cdot k_E \cdot k_F$$
(18)

Where \dot{B} = set of stresses and strains conditions; β_z = special realisation of stresses and strains conditions; $k_{\dot{B}}$ = number of realisations = number of classes.

3.2.4. Module 4: comparison parameters

The comparison parameters are necessary for the proofs. They describe the allowed load cycles concerning the realisations of the stress and strain conditions. In the course of the probability pavement design process the comparison parameters have to be assigned as random variables. The set of classified / discrete comparison parameters is defined by:

$$N = \{n_{o} \mid 1 \le o \le k_{N}\} \text{ mit } n_{o} \in [n_{o} - \frac{\Delta n_{o}}{2}, n_{o} + \frac{\Delta n_{o}}{2}]$$
(19)

Where N = set of comparison parameters; n_o = special value of comparison parameters; k_N = number of realisations = number of classes.

And for the associated probability for occurrence applies:

$$P(N \in [n_o - \frac{\Delta n_o}{2}, n_o + \frac{\Delta n_o}{2}]) = P(n_o) = \int_{n_o - \frac{\Delta n_o}{2}}^{n_o + \frac{\Delta n_o}{2}} f(n) dn \quad \text{mit} \quad \sum_o P(n_o) = 1$$
(20)

3.2.5. Module 5: damage hypothesis, total and partly damage

From each special value of stress and strain conditions β_z a partly damage s_p is following and can be calculated using the comparison parameters n_o . As previously mentioned comparison parameters describes the allowed load cycles referring at a defined strain condition β_z . The partly damage is defined by the quotient of number of load cycles applied at a value of the combined load variables b_a and the allowed load cycles at the strains condition β_z resulting from b_a and r_h . The allowed number of load cycles depends on the kind of proof (see also chapter 2). The partly damages will be calculated for every realisations of combined load variables b_a according to the RDO Asphalt 09 [1]. Furthermore the partly damages have to be calculated for all realisations of the combined characteristics r_h and all realisation of comparison parameters n_o using the probabilistic pavement design process. For every realisation applies:

$$s_{p} = s(b_{a}, r_{h}, n_{o}) = s(\beta_{z}, n_{o}, b_{a}) = \frac{\dot{N}(b_{a})}{n_{o}(\beta_{z})} = \frac{\underline{N} \cdot P(b_{a})}{n_{o}(\beta_{z})}$$
(21)

Where <u>N</u> = load cycles during the service life; \dot{N} = load cycles applied the especially realisation of the combined load variables b_a .

The total damage of the pavement will be calculated by the sum of all partly damages over all combined load variables b_a according to Miner's Law.

$$S(r_{h}, n_{o}) = S(m_{c}) = \sum_{b_{a}} s(b_{a}, r_{h}, n_{o})$$
(22)

The total damages have to be calculated for all combinations of characteristics and comparison parameters. These combinations results using the Cartesian product:

$$M = R \times N = \{m_c \mid 1 \le c \le k_M\} \text{ mit } k_M = k_R \cdot k_N$$
(23)

Where M = set of combined characteristics and comparison parameters; m_c = especially realisation of combined characteristics and comparison parameters; k_M = number of realisations = number of classes.

The set of combined characteristics and comparison parameters M in matrix notation:

$$M = \begin{cases} (r_{1}, n_{1}), & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & (r_{k_{R}}, n_{1}), \\ \vdots & \vdots \\ (r_{1}, n_{1}), & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & (r_{k_{R}}, n_{k_{N}}) \end{cases} = \begin{cases} m_{1}, & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & m_{k_{M}} \end{cases}$$
(24)

The corresponding probabilities and total damages can also be represented in matrix notation.

$$P(M) = \begin{cases} \left(P(d_{1}) \cdot P(e_{1}) \cdot P(n_{1}) \right), & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & (P(d_{k_{p}}) \cdot P(e_{k_{p}}) \cdot P(n_{k_{N}})) \right) \\ = \left\{ P(m_{1}), & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & P(m_{k_{M}}) \right\} \end{cases}$$

$$S = \begin{cases} \sum_{b_{a}} s(b_{a}, r_{1}, n_{1}), & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & \sum_{b_{a}} s(b_{a}, r_{k_{R}}, n_{1}), \\ \vdots & \cdots & \sum_{b_{a}} s(b_{a}, r_{k_{R}}, n_{k_{N}}), \\ \vdots & \cdots & \sum_{b_{a}} s(b_{a}, r_{k_{R}}, n_{k_{N}}), \end{cases} = \begin{cases} S(m_{1}) & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & S(m_{k_{M}}) \end{cases}$$
(25)
$$(25)$$

$$(26)$$

3.2.6. Module 6: Probability of failure

All proofs that are carried out in the course of the design process for asphalt pavements using Miner's Law [5]. According this law, if the sum of the partly damages over all combined load variables b_a greater than 1, than the proof is not provided. The total damage will be calculated as a sum of all combined load variables b_a (object constant random variables). Furthermore, the total damage have to be also calculated for each combination of characteristics and comparison parameters (object variable random variables) using the probability design process. Hence, a value of total damage, that is either greater than 1 (proof is not provided) or smaller and equal than 1 respectively (proof is provided), will be calculated for every realisation of combined characteristics and comparison parameters. The set of all total damages S have to be describes using a indicator $I_{(S(mc))}$ for calculating the probability of default.

$$I_{\{S(m_c)\}} = \begin{cases} 1 & f? \ S(m_c) > 1 \\ 0 & f? \ S(m_c) \le 1 \end{cases}$$

$$I_{\{S(m_c)\}} = \begin{cases} I_{\{S(m_1)\}} & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & I_{\{S(m_{k_1})\}} \end{cases}$$
(27)
(28)

Following, the product of the probability $P(m_c)$ (equation 25) and the associated indicator $I_{S(mc)}$ will be calculated for every realisation of combined characteristics and comparison parameters and added up over all these realisations. The result is the probability of default PD.

$$PD = \sum_{c} I_{\{S(m_c)\}} \cdot P(m_c)$$
(29)

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4. MODEL CALCULATION

The probabilistic pavement design process shall be exemplified by the following model calculation. In this example, the asphalt stiffness modulus, the allowed load cycles (load cycles until macro-cracking) as well as the thicknesses of the full-depth asphalt pavement have been handled as random variables. The calculations were carried out with the calibration asphalts according to the RDO Asphalt 09 [1]. The variability of the calibration asphalts will be described by normal distribution using fictive distribution parameters.

The scatters of the stiffness modulus can be described advantageously using the relation of stiffness modulus. For large undershooting probabilities the stiffness modulus can be negative depending on the standard deviation when the dispersion of the stiffness modulus about the estimated value (results of laboratory tests) will be described by the difference between measured and estimated value of the regression model. With $E \in \mathbb{R}^+$, the distribution functions have to be cut for all negative stiffness modulus E < 0. Therefore it is appropriate using the relation of stiffness instead of their difference.

$$\dot{E}_{\xi} = \frac{E_{\xi}}{\mu_{\rm E}} \tag{30}$$

Where \dot{E}_{ξ} = special value of relation of stiffness modulus [N/mm²] (with $\dot{E}_{\xi} \in [0,1]$); E_{ξ} = stiffness modulus – result of laboratory test [N/mm²] (with $E_{\xi} \in \mathbb{R}^+$); μ_E = expectancy of stiffness module [N/mm²] (with $\mu_E \in \mathbb{R}^+$).

The fatigue function of an asphalt describes the correlation between the initial elastic strains and the load cycles until macro-cracking. In the following the random variable "load cycles until macro-cracking" will be designated with N as well as their special / random value with N_§. The describtion of the variability of the load cycles until macro-cracking should be carried out with the logarithmised fatigue function advantageously.

$$\ln \mu_{\rm N} = b \cdot \ln \varepsilon_{\xi} + c \tag{31}$$

$$\Delta \ln N_{\varepsilon} = \ln N_{\varepsilon} - \ln \mu_{N}$$
(32)

Where $ln\mu_N = logarithmised$ expectancy of load cycles until macro-cracking [-] (with $\mu_N \in \mathbb{R}^+$); b, c = material parameters [-]; $ln\epsilon_{\xi}$ = special value of the logarithmised initial elastic strain (with $\epsilon_{\xi} \in \mathbb{R}^+$) [-]; lnN_{ξ} = special value of logarithmised load cycles until macro-cracking (with $N_{\xi} \in \mathbb{R}^+$) [-]; ΔlnN_{ξ} = difference of logarithmised load cycles until macro-cracking [-].

The special values ΔInN_{ξ} of the random variable "difference of logarithmised load cycles until macro-cracking" ΔInN can be stochastically modeled with a normal distribution.

Both the thicknesses of the particular construction layers and the thicknesses of the fulldepth asphalt pavement as well as the thicknesses of the pavement structure are indicated by variations. The difference of thicknesses ΔH (the thicknesses of individual layers as well as the thicknesses of full-depth asphalt pavement) can be approximated with a normal distribution [6]. That is the result from extensive statistical evaluations of thickness measurements on asphalt pavements (measurements of thicknesses of individual layers and measurements of thicknesses of full-depth asphalt pavement). The variability of thicknesses of full-depth asphalt pavement has been taken into account for the model calculation.

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Where ΔH_{ξ} = special value of the difference of thickness of full-depth asphalt pavement [mm]; H_{ξ} = special value of thickness of full-depth asphalt pavement [mm]; μ_{H} = estimate expectancy of thickness of full-depth asphalt pavement [mm].

The continuous probability density functions can be determined depending on the estimated expectancy and standard deviation for the relations of stiffness modulus, the differences of logarithmised load cycles until macro-cracking as well as the differences of thicknesses of full-depth asphalt pavement. The continuous probability density functions have to be discretised when the probabilistic pavement design process will be used. The discretisation can be carried out with integration of the continuous probability density functions of all three random variables have been discretised with nine classes per random variable (Table 1).

	C1	C2	C3	C4	C5	C6	C7	C8	C9
Ė	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20
In∆N	-2.00	-1.50	-1.00	-0.50	0.00	0.50	1.00	1.50	2.00
ΔH	-2.00	-1.50	-1.00	-0.50	0.00	0.50	1.00	1.50	2.00

The standard deviations of the random variables have been defined with $\sigma_{\dot{E}}$ =0.075, $\sigma_{\text{In}\Delta N}$ =0.75 and $\sigma_{\Delta H}$ =0.75. Figure 6 (left diagram) shows the discretise probability density function resulting from these assumptions.



Figure 6 - Discretise probability density functions; left: with σ_{E} =0.075, $\sigma_{In\Delta N}$ =0.75 and σ_{H} =0.75; right: depending standard deviations (classes of standard deviations see table 3).

The sum of all load cycles during a service life of 30 years have been defined with $\underline{N} = 150,000,000$ load cycles. This corresponds to an average daily traffic volume of 3,260 vehicles. The axle load distribution (classes of axle loads and their probability of occurrence) have been used for this model calculation correspond to the axle load distribution for long distance traffic according to RDO Asphalt 09 [1]. The characteristic temperature profiles defined by Kayser [7, 8] have been used for the model calculation deviated from the RDO Asphalt 09 [1]. Thus, 204 temperature profiles could be considered. The pavement structure have been defined so that the sum of partly damage (total damage) is 1 (S=1) using the following special value of comparison parameters and characteristics.

 $\{\dot{E} = 1.0\} \cap \{\Delta \ln N = 0.0\} \cap \{\Delta H = -2.0\}$

That is the reference configuration for the following examinations.

Table 2 - Pavement structure with informations to the layers, the thickness of layers and
material parameters of layers.

structure layer	thickness of layer	material parameters			
asphalt surface layer	4 cm	expectancy of stiffness modulu according to RDO Asphalt 09			
asphalt binder course	8 cm	(stiffness-temperature-function of asphalt surface layer, asphalt binder			
asphalt base course	22 cm	course and asphalt base course) Poisson's ratio = 0.35			
frost blanket course	56 cm	modulus of deformation = 120 N/mm ² Poisson's ratio = 0.5			
Formation	×	modulus of deformation = 45 N/mm ² Poisson's ratio = 0.5			

Figure 7 shows the total damages S (damage sum) depending on \dot{E} und ΔlnN exemplary for ΔH_{ξ} =-2 (left diagram) and ΔH_{ξ} =0 (right diagram). The probability of failure is about 31 % referring to the reference configuration when a recalibration of the probabilistic design process will be disregarded.



Figure 7 - Damage sum depending special values for the random variable \dot{E} und Δ InN; left: ΔH_{ξ} =-2.0; right: ΔH_{ξ} =0.0.

The variability of random variables will be characterised by their standard deviations. At unchanged boundary conditions, the probability of failure concerning the reference configuration will be also changed by modifying standard deviations. The standard deviations have been varied as follows in this example to using the probabilistic design process (Table 3).

Table 3 – Reference values of ever	y classes standard deviations	per random variable.
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	σ1	σ2	σ3	σ4	σ5
Ė	0.025	0.050	0.075	0.100	0.125
lnΔN	0.250	0.500	0.750	1.000	1.250
ΔH	0.250	0.500	0.750	1.000	1.250

The allocated discrete probability densities are presented in figure 6 (right diagram). Figure 8 shows the probabilities of failure for different variabilities of random variables. The random variable Δ InN (differences of logarithmised load cycles until macro-cracking) has the greatest influence on the probability of failure. The correlation between the standard deviations of the random variable Δ InN and the probabilities of failure is R=0.947. The random variable Δ H (difference of thickness of full-depth asphalt pavement) has the lowest influence on the probability of failure with a correlation coefficient R=0.0026.



Figure 8 - Probabilities of failure depending special values for the standard deviations of the random variable Δ InN and \dot{E} with $\sigma_{\Delta H}$ =0.25 (left) as well as special values for the standard deviations of the random variable Δ InN and Δ H with $\sigma_{\dot{E}}$ =0.025 (right).

5. SUMMARY

Asphalt pavements can be designed more economically and ecologically sustainable with the German guideline for the design process of asphalt pavements (RDO Asphalt 09) [1] in contrast to the German guideline for the standardisation of asphalt pavements (RStO 01) [9]. Thereby, the necessary thicknesses of structure layers results from corresponding stresses and strains into the structure. With this new guideline, the highway engineer has a tool to consider different parameters that influence the design process result and he can takes into account these parameters individually.

The design process of asphalt pavements is a significant innovation compared to the empirical procedure according RStO 01 [9], because the mechanical parameters of asphalts and their individuality among other things will be considered. The scatters of these material parameters and thicknesses of structure layers respectively will be neglected presently.

Asphalts with the same expected values concerning the corresponding relevant material parameters will be treated equally in the current design process. Pavements, that was built with these asphalts (asphalts with same expected values of their material parameters) can have different safety levels and hence also the probabilities of failure, when the scatters of the material parameters are unequal. A real judging comparison of this asphalts is not possible and only possible to a limited extent respectively. The same applies for the judging comparison of the pavement structure.

The variabilities of the relevant input parameters for the design process can be taken into account with the probabilistic design process for asphalt pavements that was described in this paper. The variabilities will be modeled by probability density functions and then they will be dicretised. The probabilistic design process for asphalt pavements enables the user to calculate the probability of failure of the pavement structure. It could be shown that the variablities of different input parameters have varying influences of the probability of failure. Now safety considerations can also be conducted with the probabilistic design process for asphalt pavements. Furthermore, different pavement structures and materials can be compared and assessed better.

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