CONSISTENCY ANALYSIS IN HORIZONTAL REVERSE CURVES

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ABSTRACT

Consistency analysis of geometric design is a useful tool to evaluate safety in roads. Nevertheless only a few geometrical design standards include recommendations for assessing consistency in horizontal alignment.

For horizontal reverse curves (or s-shaped curves), Chilean and Spanish standards establish that if tangent length is lower than 400 m, operating speeds on both curves are correlated, and a entrance/exit radius ratio is recommended for a good design. Nevertheless the relationship among operation speed, tangent length, and acceleration rates are not considered in the consistency analysis.

In this paper the consistency in reverse curves design, considering different radius ratio, design speeds, tangent lengths and accelerations is analyzed and the criterion used in Chile is verified. Three scenarios with 350 combinations of geometric designs were simulated. Simulation techniques were used for each case, by using acceleration values of 0.4 m/s2 and 0.85 m/s2 estimated in Chile and in United States, respectively.

It was concluded that the condition of tangent length of 400 m used in Chilean standard is not enough to ensure correlation between speeds on curves and a consistent design. The consistency is dependant of the combination of radius values, tangent length, design speed and acceleration rates used.

Key Words: Consistency, reverse horizontal curves, tangent length, design speed, operating speed.

1. INTRODUCTION

Guidelines of road geometrical design provide recommendations for choosing the minimum geometric elements regarding the goal of highway (mobility or accessibility) and its hierarchy. Specifically, in horizontal alignment, provide methods for selecting radius and superelevation, according to a design friction factor and design speed. However, empirical evidence has revealed several weaknesses of design methods based on design speed [1]. Krammes [2] found that in highways of design speeds lower than 90 km/h operating speeds were higher than design speed. This evidence controverts the standard conception of design speed (S_d) : "the maximum safe speed that a driver can attain in a section of road when the speed only is conditioned by the geometry conditions" [3] and shows that there exist a gap between theoretical assumptions related about design speed and operating speed (S_{op}) . This gap is caused mainly for the response of drivers to the road geometry.

McLean [4] proposed that discrepancies between design and operating speed cause inconsistencies and increases the risk of accidents. Messer [5] proposed that inconsistencies were due to an increasing of driver workload. Those proposed were the starting point for two approaches to geometric consistency analysis: speed-based consistency analysis and workload-based consistency analysis [6].

On the other hand, Lamm et al [7] founded a strong correlation between occurrence of accidents in horizontal curves and a break on driver´s expectancy of road geometry. A rupture in the driver's expectancy generates differences between operating speed and design speed. Lamm explains that a road is consistent if there is a little gap between operating speed and driver speed. In this case, road geometry is consistent with driver expectancy.

Based on in-field studies, Lamm proposed three criteria to assess consistency of road geometrical design:

- Criterion I, based on speed in single elements. It analyses the difference between operating speed and design speed in single horizontal curves.
- Criterion II, based on speeds in successive elements. It analyses the difference between operating speeds in successive horizontal curves.
- Criterion III, based on friction demand in single elements. It analyzes the difference between side friction demand and design friction in horizontal curves.

The Lamm's criteria permit to evaluate the consistency in the driver's expectancy based on speed or friction demand. For the three criteria Lamm proposes thresholds to rank geometrical design in 3 categories: "good", "fair" and "poor" [8]. The application of these criteria permits to improve the safety in the geometric design process of new roads or in existent roads when is necessary to improve its geometry.

Just a few geometrical design standards include explicitly recommendations and methods for assessing consistency. For instance, South African [9], Canadian [10], German [11], New Zealand [12] and Australian [13] standards includes operation speed models that permits to compare it with design speed in isolated horizontal curves (Criterion I). Only Canadian, German and South African standards provide recommendations for the consistency assessment of successive curves (Criterion II).

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The Chilean standard establish that reverse curves (s-shape curves) with length of tangent lower than 400 m are dependant one of each other and an adequate selection of the ratio entrance/exit radius is necessary for a safety condition [14]. However based on a consistency framework, this criterion should be verified because the operating speeds in reverse curves are depending of the radius, the tangent length and the acceleration/deceleration rates in tangent section.

2. OBJECTIVE AND SCOPE

The objective in this paper is to analyze the consistency in reverse curves and to verify the criterion used in Chile to design reverse curves. For this, different entrance radius/exit radius ratio, design speeds, tangent lengths and accelerations/deceleration rates were considered.

To achieve the objectives four main steps were followed:

- Main Concepts Review. On this step, operation speed models and consistency concepts in horizontal reverse curves were analyzed. Particularly Lamm's criterion II related to consistency between successive curves was studied.
- Analysis of reverse horizontal curves design guidelines. Several geometric design guidelines were assessed to summarize the main aspects to design reverse curves. Guidelines from Unites States, Canada, Germany, Spain, Swiss, Chile, Colombia, South Africa and Australia were studied.
- Simulation of sceneries. Three sceneries were analyzed: entrance radius higher, equal or lower than the exit radius. Design speed between 40 km/h and 80 km/h and accelerations rates of 0.85 m/s2 and 0.40 m/s2 were considered. A total of 350 cases were simulated and for each case the interaction between entrance and exit radius and its relation with tangent length and consistency was studied.
- Verification of Chilean criterion to design reverse curve. The criterion of Chilean standard for designing reverse curves was discussed and final recommendations were proposed.

3. THEORETICAL FRAMEWORK

3.1 Speed Concepts

Speed is the main variable used in consistency analysis. Design speed (Sd) and operation speed (Sop) are considered in Lamm's criterion I and II.

Design speed concept has changed from the first concept proposed by Barnett in 40's [15] to a modern concept proposed by AASHTO [16]. The AASHTO Green Book of 1994 defines: "design speed is the maximum safe speed that can be maintained over a specified section of highway when conditions are so favorable that the design features of the highway govern" [17] while the AASHTO 2004 defines design speed as "a selected speed used to determine the various geometric design features of the roadway" [18].

The New Zealand standard defines design speed as synonymous of operating speed. This standard assumes that estimated 85th percentile of speed distribution does not exceed design speed, and recommend the selection of design speed according to speed environment. Speed environment and design speed are measured as the 85th percentile of speed distribution, observed in longer straights or large radius curves when traffic volume is very low. In the case of British standard design speed is obtained from the restrictions of sight distance, points of accesses by kilometer and the maximum legal speed [19]. This standard does not incorporate consistency and operation speed concepts.

Operating speed is the speed that the drivers are observed operating its vehicles [20]. A common descriptor of operating speed is the percentile 85 of speed distribution. The operation speed can be represented by an operating speed profile which includes in the xaxis path length and in the y-axis the operation speed. Operation speed profile permits to appreciate the magnitude of the differences of Sop between successive elements in the road.

A main aspect in consistency assessment is to modelate operating speed. Several models have been calibrated since 1950, and today there are approximately 130 operating speed models. A few geometrical design standard use operating speed models for design [21] (For instance, German, Swiss, South Africa and Canadian standards). Particularly, on reverse horizontal curves two types of models can be useful to estimate an operating speed along the curve – tangent – curve element: operating speed models in single horizontal curves, and operating speed on tangent. In this paper the first type of model is considered, according to local calibration.

3.2 Consistency in Reverse Horizontal Curves Concepts

Reverse horizontal curves (or s-shape curves) is a geometrical configuration composed by an entrance curve (R1) and an exit curve (R2) joined by a transition tangent with opposed deflection angles. Operating speeds in reverse horizontal curves are depending of the radius of each curve and of tangent length. If tangent length is higher, drivers can achieve operating speed in tangent near to desired speed. In this case, the operating speed of the exit curve is independent of the operating speed on entrance curve. Both curves operate as isolated horizontal curve. In contrast, whether tangent length is lower, drivers accelerate or decelerate to negotiate approach speed to the exit curve. In this case, driver' speed on tangent is correlated to both entrance and exit curves.

Figure 1 shows different operating speed profiles where R2 > R1. Profile 1 shows the case in which tangent length is lower than a minimum value (TLmin) and the driver accelerate from operating speed in entrance curve (V85,1) to achieve the operating speed in the exit curve (V85,2).. Profile 2 shows the case in which the tangent length is higher than a maximum value (TLmax) and drivers can achieve the desired speed in the tangent section (V85,TMax) . In this configuration drivers accelerates from the speed on entrance curve (V85,1) to desired speed in the tangent (V85,TMax), keep this speed for a few seconds and decelerates from the desired speed in tangent to operating speed in the exit curve (V85,2).Operating speeds in both curves are independent and the curves operate like single curves. Finally, profile 3 shows the case in which drivers accelerate from operating speed in entrance curve (V85,1) to a speed lower than desired speed in tangent but higher than the operating speed in exit curve. Then, drivers must decelerate to reach operating speed on exit curve (V85,2).

It is relevant to recognize this behavior and the configurations of speed profiles because define how consistency assessment should be applied in the analysis.

a) Operating Speed Profile 1 when $TL < T_{Lmin}$

b) Operating Speed Profile 2 when $TL > TL_{max}$

c) Operating Speed Profile 3 when TL_{min} < TL < TL_{max}

Figure 1 - Theoretical operating speed profiles in reverse horizontal curves for $R_1 < R_2$

To discriminate each case, Lamm et al [22] proposed an easy method based on minimum and maximum tangent length concepts. He postulates that reverse horizontal curves are correlated one to each other if tangent length is between a minimum and a maximum tangent length value.

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Minimum tangent length (TLmin) is defined as the length that drivers needs to accelerate (or decelerate) from a starting operating speed (V85,1) to an exit operating speed (V85,2). TLmin can be calculated according to Eq. 1, where "a" is the acceleration (or deceleration) in m/s2. (0.4 m/s2 to 0.85 m/s2). If both radius are equals operating speed are the same an TLmin is zero.

$$
TL_{\min} = \frac{|V_{85,1}^2 - V_{85,2}^2|}{25.92a}
$$
 (1)

Maximum tangent length (TLmax) is defined as the length that driver needs to accelerate from a starting operating speed (V85,1) to a maximum operating speed (V85,TMAX) into the tangent. It can be estimated by using Eq. 2.

$$
TL_{\max} = \frac{\left|2V_{85, TMAX}^2 - V_{85, 1}^2 - V_{85, 2}^2\right|}{25.92a}
$$
 (2)

Maximum operating speed (V85,TMAX) can be estimated using speed models and considering very low curvature rate [23] [24] [25].

When tangent length is between TLmin and TLmax, drivers cannot achieve V85,TMAX. In this case, Eq. 3 can be used to estimate V85 in tangent. This model is valid if tangent length (TL) is between TLmin and TLmax or if TL is lower than TLmin and V85,1 > V85,2. Otherwise, V85,1 and V85,2 should be interchanged in Eq. 3.

$$
V_{85\,\text{max}} = \sqrt{12.04a(TL - TL_{\text{min}}) + V_{85,1}^2}
$$
 (3)

However, the Lamm's method does not consider the autocorrelation effect between operation speeds on curves. For this reason, the use of in-field calibrated models that relate driver behavior in tangents with the speed on exit curves is recommended.

According to the Lamm's method the following cases can be identified when tangent length and Lamm's threshold are compared [26]:

- Case 1: $TL \leq TLmin$: Both curves are dependant. The sequence entrance curve to - exit curve is relevant for assessing consistency..
- Case 2: TL \ge TLmax: Both curves are independent. The sequence tangent-to- exit curve is relevant for assessing consistency. Probably desired speed is achieved in the tangent.
- Case 3: TLmax \leq TL \leq TLmin: Both curves are isolated and the sequence tangentto- exit curve is relevant for assessing consistency. Probably tangent speed is lower than desired speed, and can be estimated by using Eq. 3.

3.3 Consistency Criterion in Reverse Horizontal Curves

For reverse curves Lamm et al [26] proposed the difference between entrance and exit operating speed as a consistency index (See Eq. 4). This is known as "Criterion II: consistency in successive design elements".

$$
\Delta V85 = |V_{85,1} - V_{85,2}| \tag{4}
$$

The criteria for consistency assessment for this type of horizontal alignments are (Lamm, 2007):

Good design (∆V85 < 10 Km/h). The difference of operating speeds does not induce alignment inconsistencies. There is consistency in the design of successive elements.

Fair design (10 Km/h ≤ ∆V85 < 20 Km/h). The difference of operating speeds can induce little alignment inconsistencies. To redesign is not recommended but probably signposting be necessary.

Poor design (∧V85 ≥ 20 Km/h). The difference of operating speeds can induce inconsistencies in the alignment. To redesign is strongly recommended.

4. REVERSE HORIZONTAL CURVES IN ROAD GEOMETRIC DESIGN GUIDELINES

Geometric design guidelines usually consider two aspects for designing reverse curves: tangent length and entrance/exit radius relationships. Exceptionally, a few guidelines consider the Lamm's Criterion II to the consistency assessment. In this section, a comprehensive discussion about these aspects is performed.

4.1 Tangent Length

Generally design standards provide controls for minimum and maximum tangent length based on driving time in the curve-tangent-curve configuration and the superelevation transition that the tangent length permits. For instance, Table 1 shows minimum tangent length allowed in Chilean standards on reverse curves.

Table 1. Project Speed and Minimum Geometric Conditions of Chilean Standards

Swiss and German standards estimate tangent length according to acceleration rate. Swiss standard consider an acceleration of 0.8 m/s2, while German standard uses 0.85 m/s2. Both standards consider tangent length as an independent design element and it is limited considering the difference between maximum operating speeds on curves [27]. Australian standard uses a similar criterion, but including acceleration in tangent as an input in a operating speed model.

4.2 Entrance/Exit Radius Relationships

Limitations to entrance and exit radius on reverse curves can be found in German, Swiss and Spanish standards. German standard establish minimum and maximum entrance / exit radius ratio considering Lamm's criterion II and the consistency limits [28]. Spanish

standard [29] considers that entrance and exit curves are composed and dependants if tangent length is lower than 400 m. In this case, the Eq. 5 is used to estimate R1/R2 ratio when R1 ranges between 50 and 300 m. This criterion is showed in Figure 2 which is the design abacus in the Chilean standard.

Figure 2 - Relationship between entrance and exit radii on reverse horizontal curves

5. CASE STUDY: CHILEAN DESIGN GUIDELINES FOR REVERSE HORIZONTAL CURVES

Case study was applied to the reverse horizontal curves design in two-lane roads. For different configurations of entrance/exit radius and tangent length the dependence between the curves was verified (single curves or composed curves) and in the case of composed curves criterion II was applied to evaluate the consistency.

Chilean standard defines project speed (Sp) as the speed that permits to define the minimum geometric characteristics of roadways. For this case project speed and design speed are analogous. For each project speed the Chilean standard establishes a minimum tangent length on reverse curve and a minimum radius on single horizontal curves (See Table 1). Similarly to Spanish standards, establish relationships between entrance and exit radius, as was showed in Figure 2 and Eq. 5.

Considering these recommendations, three scenarios were simulated: R1 = R2, R1 > R2 and R1 < R2. Project speed between 40 km/h and 80 km/h and different tangent length were considered. A total of 350 combinations of geometric designs were simulated and analyzed. Lamm´s criterion was used to evaluate the dependence between the curves and the consistency for different tangent length in each scenario.

To estimate operating speed (V85) in horizontal curves, Saez´s model for single horizontal curve was used [30]. This model was calibrated for Chilean two-lane rural roads (See Eq. 6).

$$
V_{85} = 95 - \frac{1880}{R}
$$
 (6)

When radius (R) tends to a high value, V85 is an estimator of maximum theoretical speed in the tangent section (V85,TMAX). In this case the maximum speed on tangent (R→∞) would be 95 km/h.

To estimate TLmin, TLmax and operating speed (S_{op}) in tangent, two values of accelerations were used: 0.85 m/s2, recommended in German standard, and 0.4 m/s2, calibrated in Chile by Echaveguren and Basualto [31]. Table 2a and 2b shows simulation results for each acceleration value. For each project speed and geometric configuration the critical values of TLmin and TLmax are calculated using eq. 1 and 2. For each configuration a classification of reverse curve as single (S) or composed (C) is given. This classification is obtained from the comparison between tangent length used in design (TL) and the minimum (TLmin) and maximum (TLmax) tangent lengths obtained by using Eq. 1 and 2 respectively.

Table 2a - Classification of Reverse Horizontal Curves for a = 0.85 m/s² (cont...)

S: Single; C: Composed

f

Table 2b - Classification of Reverse Horizontal Curves for $a = 0.40$ m/s².

Consistency criterion II was applied to the configurations of Table 2a and 2b to evaluate consistency in geometric design. Results are summarized on Table 3a and 3b.

G: Good Design; F: Fair Design; P: Poor Design

	$Sp = 40 (Km/h)$		$Sp = 50$ (Km/h)		$Sp = 60$ (Km/h)		$Sp = 70$ (Km/h)		$Sp = 80$ (Km/h)	
Configuration	TL(m)	Class	TL(m)	Class	TL(m)	Class	TL(m)	Class	TL(m)	Class
	55	G								
$C1: R_1 = R_2$	$70\,$	G	70	G						
	85	G	85	G	85	G				
	100	G	100	G	100	G	100	G		
	115	G	115	G	115	G	115	G	115	G
	200	G	200	G	200	G	200	G	200	G
	300	G	300	G	300	G	300	G	300	G
	400	G	400	G	400	G	400	G	400	G
$C2: R_1 < R_2$	55	G								
	70	G	70	G						
	85	G	85	G	85	G				
	100	G	100	G	100	G	100	G		
	115	G	115	G	115	G	115	G	115	G
	200	G	200	G	200	G	200	G	200	${\mathsf G}$
	300	G	300	G	300	G	300	G	300	G
	400	G	400	G	400	G	400	G	400	${\mathsf G}$
$C3: R_1 > R_2$	55	$\mathsf F$								
	$70\,$	$\mathsf F$	$70\,$	G						
	85	$\mathsf F$	85	G	85	G				
	100	$\mathsf F$	100	G	100	G	100	G		
	115	$\mathsf F$	115	G	115	G	115	G	115	${\mathsf G}$
	200	$\mathsf F$	200	G	200	G	200	G	200	${\mathsf G}$
	300	$\mathsf F$	300	G	300	G	300	G	300	G
	400	F	400	G	400	G	400	G	400	G

Table 3b - Consistency Assessment of Reverse Horizontal Curves for a = 0.40 m/s².

G: Good Design; F: Fair Design; P: Poor Design

6. RESULTS DISCUSSION

Results are discussed from two points of view. One related to the effect of tangent length on speed behavior in reverse horizontal curves and other related to consistency assessment of reverse curves by using Lamm's criteria.

6.1. Effect of Tangent Length on Speed in Reverse Curves

If the reverse curves are composed drivers choice the speed considering both curves and tangent length as a whole. In this case speeds on both curves (V85,1 and V85,2) are correlated. Otherwise, if horizontal curves are independent drivers can increase its speed to desired speed in certain segment of tangent length therefore the speeds on both curves are not correlated.

The results of tangent length (TLmax, TLmin and TL) obtained in the simulation process are showed in Table 2a and 2b for acceleration rates of 0.85 m/s2 and 0.40 m/s2 respectively. The results indicate that the criterion proposed in Chilean design standard is valid only for certain configuration and is depending on the project speed and acceleration rates.

When acceleration rate is 0.85 m/s2 only for project speeds lower than 45 km/h tangent length lower that 400 m induces interaction between entrance and exit curves and both curves could be considered as composed curves. When an acceleration of 0.4 m/s2 is used, the same condition is accomplished when project speed is lower than 65 km/h, approximately. A graphic representation about the relation between project speed, acceleration rates and TLmax is showed in Figure 3.

Figure 3 - Relation among project speed, acceleration rates and TL_{max}.

6.2. Consistency Assessment of Reverse Curves

The consistency assessment was carried out for all configurations of reverse curves, combined or singles. In the case of independent curves (TL > TLmax) Lamm's criterion I was applied. The results were showed in Table 3a and 3b respectively.

The results shows that there is no relation between type of configuration (singles or combined) and the consistency. Consistent (or inconsistent) designs can be found in singled and composed configuration. For a = 0.85 m/s2, "fair" and "poor" design are found in the case of single curves when project speed ranges between 50 and 60 Km/h, for all radius configuration, and in the case of combined curves when project speed is 40 km/h and the radius configuration is $R1 > R2$. For a = 0.40 m/s, "fair" design are found only for combined curves with project speed of 40 km/h and radius configuration of R1 > R2.

In the case of single curves configuration and low project speed, tangent length (TL) permits to drivers accelerate to a maximum speed on tangent (V85,Tmax), but the design speed in the exit curve is lower (V85,2). The difference between speed on tangent and speed on exit curve (or entrance) generate the inconsistency. In this case, an improvement of design is obtained diminishing tangent length to a value that permits interaction between both curves (combined curves) and avoid that the drivers reach high speeds in the tangent.

For combined curves with configuration $R1 > R2$ and low project design the effect of the entrance radius (R1), enlarge the effect of tangent length on the consistency. This effect is higher for low project speed because the design of R2 is more restrictive (minimum radius).

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Another key aspect is the effect of acceleration on the results obtained. When locally calibrated acceleration of 0.40 m/s2 is used, only a few cases of "fair" design are founded. In the most cases the design are "good", including the single curves configuration. This is because local driver need more acceleration (or deceleration) distance and the probability that they increase speed to maximum speed on tangent is low. Therefore, is very important to calibrate locally acceleration values for apply consistency criterion.

Finally, the control over the tangent length in successive curves is not enough to achieve the consistency in the design. It is necessary consider operating speed models and calibrated acceleration rates to evaluate the type of configuration (single or composed) and the design consistency.

CONCLUSIONS

For reverse horizontal curves design the most of geometric design standards only includes design recommendations based on travel time along tangent segment. This criterion is related mainly with driver workload on this type of segment.

Operating speeds models and consistency assessment are not incorporates in the most of design standards. Exceptions are Canadian, South African and German Standard that includes explicitly consistency assessment; also, Australian and Swiss standards, uses operating speed models that permit a better estimation of speed profile and a better control of differences in operating speed between successive elements. Other standards like Chilean and Spanish, only defines limitations to entrance / exit radius ratio and assumes that if tangent length is lower than 400 m both curves are related one to each other and the geometric configuration corresponds to composed curves.

In this paper the consistency in reverse horizontal curves, considering different radius ratio, design speeds, tangent lengths and accelerations rates was evaluated and the criterion used in Chile for design reverse curves was verified. Results were discussed from two points of view. One related to the effect of tangent length on speed behavior in reverse horizontal curves and other related to consistency assessment of reverse curves by using Lamm's criteria.

It was concluded that the condition of maximum tangent length of 400 m used in Chilean standard is not enough for ensuring that s-shaped curves acts as combined curves. At the same time, this criterion does not ensure consistency because this is depending of entrance/exit radius ratio, tangent length, and acceleration rates used. If designer considers the minimum tangent length of Chilean standard jointly with entrance / exit radius ratio, designs will be consistent for project speed higher than 60 km/h and accelerations of 0.85 m/s2 or higher than 50 km/h and acceleration rate of 0.4 m/s2.

A main aspect to assess consistency in reverse horizontal curves is to estimate the speed profile, and particularly the estimation of operating speed in tangent section. In this paper, an extension of operating speed model calibrated in Chile was used to estimate operating speed in reverse curves. However, a better model that includes the effect of entrance tangent length and tangent length between curves is needed, to consider speed autocorrelation and accelerations in long tangents. Further research is needed in Chile to calibrate that type of speeds models.

Finally, it is convenient that standards include recommendations about minimum (TLmin) and maximum (TLmax) tangents lengths in reverse horizontal curves, according to a consistency analysis that incorporates calibrated speed and acceleration models according to local conditions.

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